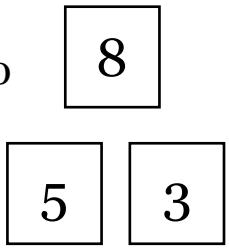
Pyramid Puzzles

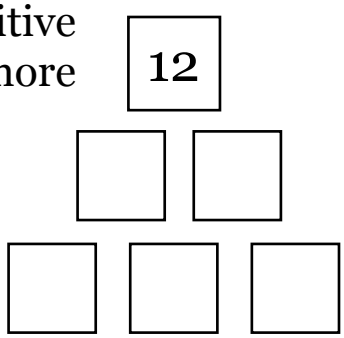
We solve pyramid puzzles by putting numbers into the empty spots so that every number is the sum of the two directly below it. A simple pyramid is just like a fact family: 8 = 5 + 3.



The goal in these puzzles is to find *nice* solutions. For a solution to be nice, it should:

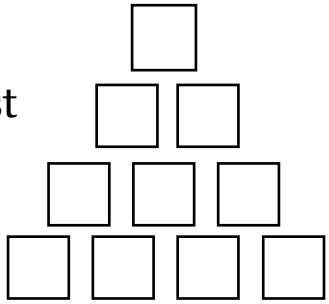
—include only positive whole numbers
—not include any number more than once

1. Solve the pyramid *nicely*, i.e., using only positive whole numbers, and not using any number more than once.



2. The smallest number that can be on top of a two-layer pyramid is 3. What is the smallest number that could be atop a pyramid with three layers?

3. What is the smallest number that can be atop a pyramid with four layers?



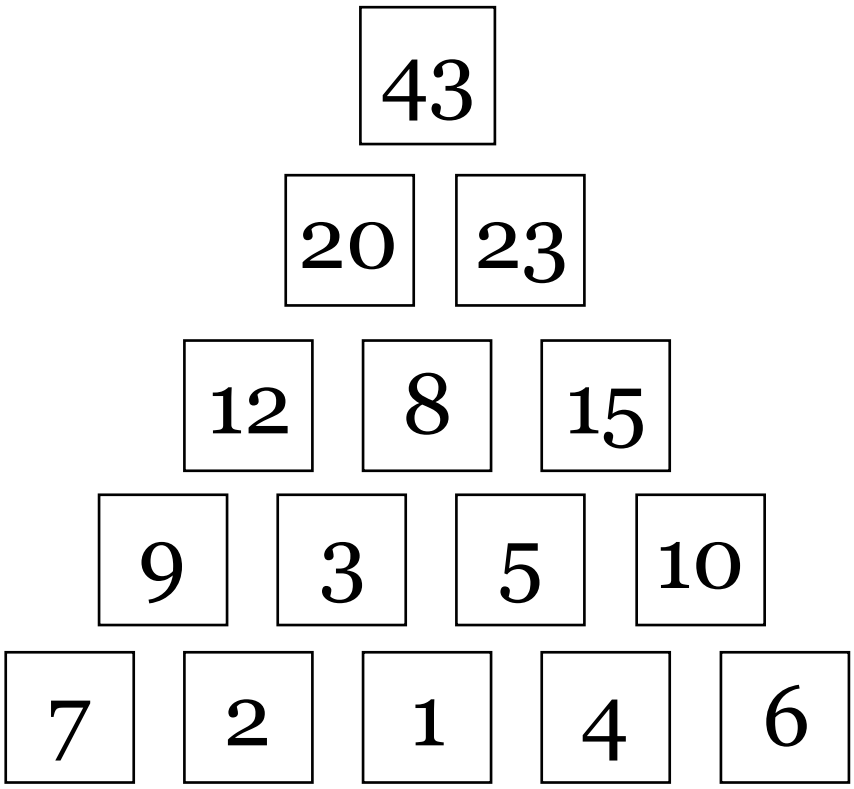
4. Continue the exploration. What is the smallest number that can be atop a pyramid with five layers? Six layers? Is there some pattern here?

5. Come up with an estimation for the minimum value the number atop a pyramid must have. Can you find a minimum bound for this value? Is there a pattern that allows you to find a good bound for any size pyramid?

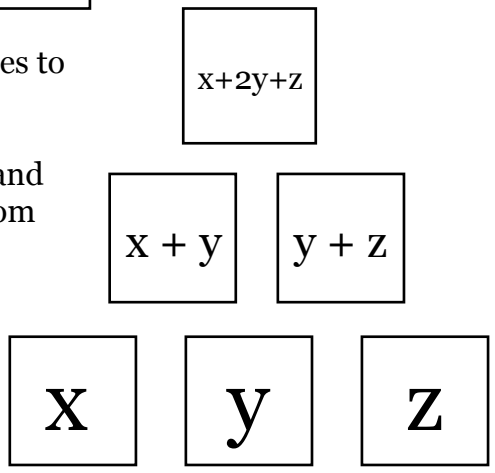
Solutions and notes for the table leader

While students can and should use paper for these puzzles, having numbers written on tokens that students can use to actually move around may be helpful for them.

The exact solution for a 5-layer pyramid puzzle is given below.



Putting variables in the bottom row can connect pyramid puzzles to Pascal's triangle.



To find estimates, you might allow repetition in higher layers, and predict what will happen if you put numbers in just in the bottom row:
1 2 for 2 layers gives 3 at the top
2 1 3 for 3 layers gives 7 at the top (8 is the actual minimum)
4 2 1 3 for 4 layers gives 16 at the top (20 is the actual min)
5 3 1 2 4 for 5 layers gives 35 at the top (43 is the actual min)

There's plenty of space to find patterns here.