Washington Office of Superintendent of
PUBLIC INSTRUCTION

INITIAL WASHINGTON STATE K–12 LEARNING STANDARDS FOR MATHEMATICS

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INTRODUCTION

The Washington State K-12 Learning Standards for Mathematics (WA Math 2025) have retained the integrity of the Common Core State Standards for Mathematics while providing clarity and access to multiple standards at the same time through their interconnection. Changes to the Common Core embrace multiple ways students demonstrate what they know and what they bring to mathematics learning. In this way, students have opportunities to engage more directly with the Common Core Standards for Mathematical Practice and assess the reasonableness of their work with respect to the questions they are seeking to explore or answer.

Key Goals of the Revisions

Revisions were guided by the following goals:

- **Structure and integrity**—Support student learning progressions and educator access to nationally aligned resources to support high quality mathematics instruction.
- **Data Science**—Ensure students can collect, analyze, understand, and critique data in a technologically data-driven world.
- **Uplift**—Center the Standards for Mathematical Practice to encourage multiple ways of thinking about and doing mathematics and for students to see the value of mathematics in their lives.
- **Clarity**—Shift to "flexibly, efficiently, and accurately" to provide clarity in what it means to be mathematically fluent.
- **Determine**—Clearly identify the content included in the first two credits of high school mathematics.
- **Prioritize**—Clearly identify standards to ensure all students have equitable access to the knowledge and skills that can be leveraged, both in school and beyond.

Structure and Integrity of the Standards

The WA Math 2025 embody the opportunity to see and explore connections within and across grades through the reorganization of the standards within Data Analysis, Quantities, Relationships, and Spatial Reasoning. Within the new structure of 4 domains, districts will have the ability to align instructional materials to the Common Core as the root coding within the original standards remains. For example, **5.MD.B.2**, a Measurement and Data standard concerning line plots of data sets is now **M.5.DA.MD.2** to reflect its interconnection with other standards within Data Analysis (DA).

Inclusion of Data Science Standards

Data science standards have been added to all grades kindergarten through high school. In a world where industries and communities rely heavily on data, students must be able to interpret, analyze, and make decisions based on reliable information. The data science standards:

- Build from the Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) framework (American Statistical Association).
- Connect math content—algebra, geometry, fractions, measurement, statistics—to real-

world inquiries.
- Encourage cross-disciplinary connections (social studies, science, English Language Arts, John McCoy (lulilas) Since Time Immemorial).

For example, students might analyze historical and current water access in relation to population growth, agriculture, and ecosystems—using municipal data, climate records, and rates of land use change.

Uplifting the Standards for Mathematical Practice

Revisions have provided the opportunity to elevate the Standards for Mathematical Practice through interconnected mathematics standards and data science inquiries. Throughout the WA Math 2025, students are encouraged to utilize multiple ways of thinking and doing mathematics, and to reflect on the reasonableness of their answers. Focusing on these practices increases students' understanding of the concepts offered in the early grades for greater success in later grades.

An example of this shift can be found in a move from "the standard algorithm" toward "a strategy or algorithm" a move that centers the many ways to efficiently solve a problem mathematically.

Both of these shifts are shown in this 6th grade standard:

- Old **6.NS.B.3**: "Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation."
- New **M.6.Q.NS.3**: "Efficiently, flexibly, and accurately add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms for each operation."

Providing Clarity

Opportunities to provide clarity in the WA Math 2025 can be found throughout the grade levels.

One example of this shift can be found in a move to clarify "fluently" to "efficiently, flexibly, and accurately". This means students can use a variety of approaches or researched strategies that work toward a solution in a way that is efficient and works toward a correct solution for different problem types. This approach provides students with strategies that can grow across grades and supports flexible mathematical thinking for a wide variety of contexts and problems. The 6th grade standard in the previous section illustrates this clarity.

Another example of providing clarity is the shift away from "from memory." This wording, as originally written in the Common Core, was not intended to be speed-based repetition of facts, and the new language supports the shift toward understanding.

An example of this shift is shown in this 3rd grade standard:

- Old **3.OA.C.7**: "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all

products of two one-digit numbers."

- New **M.3.Q.OA.7**: "Efficiently, flexibly, and accurately multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.

Priority Learning Standards

Priority Learning Standards are the most essential academic skills and concepts students need to succeed from one grade level to the next. These standards were selected to ensure all students have equitable access to the knowledge and skills that can be leveraged, both in school and beyond. Other learning standards serve to support and reinforce the Priority Learning Standards. The Priority Learning Standards identify the universal set of standards that each and every student should have the opportunity to learn and be able to do.

Because state standards encompass many learning goals for each subject and grade, OSPI collaborated with groups of educators from kindergarten through 12th grade over two years to identify the standards that are most critical to focus on in teaching and assessment. The process was guided by national best practices. Educators had multiple opportunities to review and give feedback before the Priority Learning Standards were finalized.

Priority Learning Standards were identified using the following criteria:

- **Endurance**: Will this skill or knowledge still be valuable beyond a single test, school year, or post-high school?
- **Leverage**: Is it useful across multiple subjects (for example, writing or critical thinking)?
- **Readiness**: Does it prepare students for success in the next grade level, course, or post-high school option?

Determining Standards at the High School Level

High school content standards in mathematics have been revised to more clearly show the math learning all students should engage in by the time they complete their second credit of mathematics. This is demonstrated more specifically in Algebra and Functions standards that previously addressed content that pertained to all families of functions. The standards have been revised to clarify that the first two years of high school math should include linear, exponential, and quadratic families of functions, while additional functions can be approached in a student's third credit of high school math aligned with their High School and Beyond Plan.

Additionally, to be explicitly aligned to state law guiding high school graduation requirements or equivalencies (RCW 28A.230.090 and WAC 180-51-068) the high school standards have been broken out to parallel locally determined high school math sequences. The Office of Superintendent of Public Instruction (OSPI) recognizes that school districts may choose different curricula/adopted instructional materials and some additional content (for example, absolute value functions or completing the square with quadratic functions) may be present in the first two credits of math. While there is locally determined flexibility of how and when the standards are addressed in the first two credits of high school math, the standards listed for Algebra 1 and Geometry,

Integrated Math 1 and Integrated Math 2, represent the math content all students should engage with before their 3rd credit of high school math.

Domain Revisions

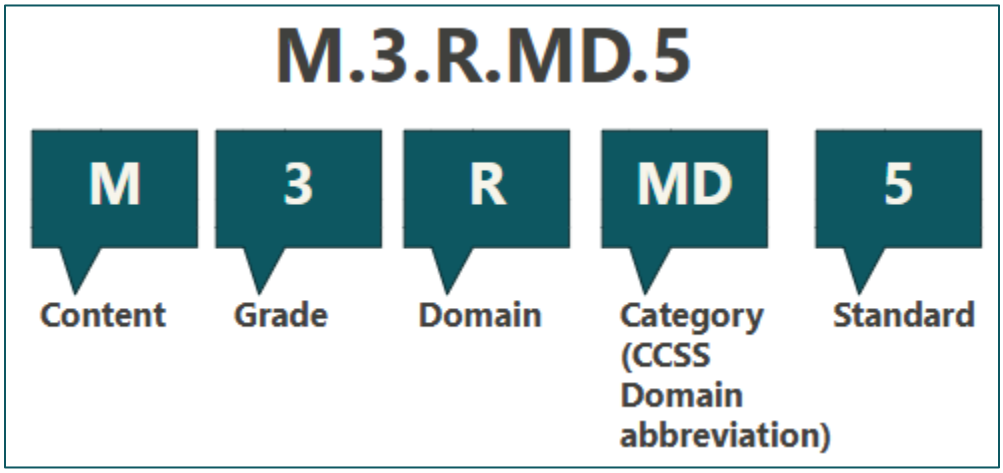
Table 1: Domain Revisions

CCSS MATH (2011) DOMAINS and CONCEPTUAL CATEGORIES	WA MATH 2025 DOMAINS	WA MATH 2025 CATEGORIES
Broad categories that grouped related mathematical content together aligned to areas of mathematics as an academic study.	Larger groups of standards that highlight their interconnected nature through broad understandings, aligned to application in real-world contexts.	Groups of standards with codes that align to the original Common Core domains to assist connections to the original Common Core standards. This will help present reviews and future adoptions of high-quality instructional materials.
Thirty-six Domains across K-12 mathematics 6 – Elementary domains 6 – Middle School domains 24 – High School domains	Four Domains that are the same across all grades in K-12 mathematics (See Key Features of Domains)	The root numbering within the mathematics standards (grounded in the original 36 domains and conceptual categories) has been carried forward in the WA Math 2025 standards as categories to maintain alignment of resources for districts.
Number of standards: 342	Number of standards: 381	Priorities: 134
Standards By Grade: K: 22 1: 21 2: 26 3: 25 4: 28 5: 26 6: 29 7: 24 8: 28 HS: 157	Standards By Grade: K: 26 1: 25 2: 30 3: 29 4: 32 5: 30 6: 33 7: 28 8: 32 HS: 160	Priorities By Grade: K: 12 1: 10 2: 13 3: 11 4: 13 5: 13 6: 11 7: 9 8: 10 HS: 32

How to Read the Standards

The WA Math 2025 expands on the structure of the Common Core State Standards for Mathematics. The order of the standards, domains, and categories within a grade level do not indicate the order in which they should be taught.

Figure 1: Math code example



Content code tells which content area or subject the standard is for. The WA Math 2025 standards use "M".

Grade level or grade band tells which grade level(s) the mathematics standard is for:

- Kindergarten uses "K".
- Grades 1 through 8 use the number for the grade level.
- High school standards use "HS".

Domains are larger groups of standards that highlight their interconnected nature through broad understandings of:

- Data Analysis "DA"
- Quantities "Q"
- Relationships "R"
- Spatial Reasoning "SR"

Categories are groups of standards which align with the original Common Core domains and cluster headings in K-8 (e.g. 2025 WA Math Category of OA comes from the Common Core domain Operations and Algebraic Thinking) and conceptual categories and domains in high school (e.g. 2025 WA Math Category of ASSE comes from Common Core HS Conceptual Category Algebra (A) and Seeing Structure in Expressions domain (SSE)).

For example, 2nd grade standards in Spatial Reasoning are grouped into the following categories:

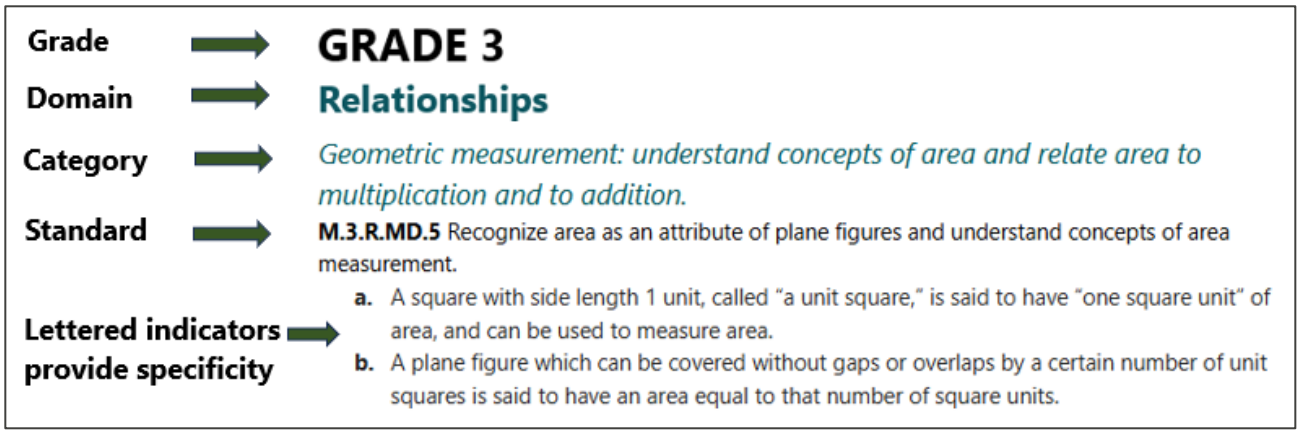
- Geometry (G): Reason with shapes and their attributes.
- Measurement and Data (MD): Work with time and money.
- Operations and Algebraic Thinking (OA): Work with equal groups of objects to gain foundations for multiplication.

Standards define what students should understand and be able to do, and have a number from 1

to 13 in the code.

Lettered indicators provide specificity for the numbered standards. Some standards have these indicators, others do not.

Figure 2: Example within Word Document



Key Features of Each Domain

WA Math 2025 is reorganized into four broad domains. This reorganization makes it easier to see connections between mathematical concepts and real-world applications. Each domain is designed to promote critical thinking, problem solving, and meaningful engagement with mathematics. The placement of a standard in a particular domain does not limit its relevance or alignment to other domains.

Data Analysis

Math standards that involve formulating statistical investigative questions, collecting and considering data including measurements, creating data visualizations (ranging from bar graphs and histograms to scatterplots and complex representations as appropriate for the grade), and interpreting results and communicating justification for conclusions.

Quantity

Math standards that involve understanding and comparing quantities through mathematical operations. This starts with analysis of magnitude and comparison of quantities in the youngest grades (including math operations and symbols) and mathematical fluency using efficient, flexible, and accurate strategies. This moves towards using math operations for place value, decimal and fractional reasoning in middle and late elementary school. Quantity culminates with examining number systems (real, integers, rational, irrational, etc.) in middle and high school grades.

Relationships

Math standards that develop strategic reasoning strategies and understanding of quantities as parts of a relationship, and interpreting and communicating about the relationship in context.

Standards in this domain connect to concepts in Data Analysis, Quantity, and Spatial Reasoning. In elementary school students examine relationships through identifying parts and wholes, using mathematical fluency strategies to perform operations with whole numbers, fractions, and decimals, connect decomposing and recomposing number strategies to shapes and fractions, then to geometry principals. Middle school students examine the relationships between parts to expressions and equations, proportionality, and set the groundwork for functions and abstract algebraic relationships in high school. Mathematical modeling of real-world relationships and problem solving are embedded within relational mathematical understanding across all grades.

Spatial Reasoning

Math standards that involve understanding shape and spread, both in geometry and data analysis (graphing). Standards in this domain also connect to Relationships, Quantity, and Data Analysis. In elementary grades students define and examine attributes of shapes and geometric principles, and the quantities/measurements associated with those attributes. Middle school students extend their understanding within graphing relationships and geometric principles in the coordinate plane. High school students examine complex attributes of geometry, foundational trigonometric principles, and graphing real world concepts to examine the meaning of shape and spread of data or a function in context.

KINDERGARTEN

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.K.DA.DS.1 Generate questions to investigate situations within the classroom.

Collect and consider data.

Priority M.K.DA.DS.2

Collect or consider data through organizing objects or drawing pictures to represent and communicate observations.

Analyze the data.

Priority M.K.DA.DS.3

Analyze data sets by noticing and describing patterns in data-rich situations.

Interpret results.

M.K.DA.DS.4 Interpret and communicate results through structured answers with teacher guidance.

Describe and compare measurable attributes.

M.K.DA.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

M.K.DA.MD.2 Directly compare two objects with a measurable attribute in common to see which object has "more of"/"less of" the attribute and describe the difference.

Classify objects and count the number of objects in each category.

M.K.DA.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

Quantity

Know number names and the count sequence.

Priority M.K.Q.CC.1

Count to 100 by ones and by tens.

M.K.Q.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Priority M.K.Q.CC.3

Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

Priority M.K.Q.CC.4

Understand the relationship between numbers and quantities; connect counting to the number of items in the set.

- **a.** When counting objects, use number names in the correct order, matching each number to exactly one object and each object to exactly one number.
- **b.** Understand that the last number name said represents the number of objects counted, and that this total stays the same no matter how the objects are arranged or the order in which they are counted.
- **c.** Understand that each successive number name refers to a quantity that is one larger.

Priority M.K.Q.CC.5

Count to answer "how many?" questions about as many as 20 objects arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

Priority M.K.Q.CC.6

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.

M.K.Q.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Relationships

Represent and solve problems involving addition and subtraction.

M.K.R.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Priority M.K.R.OA.2

Efficiently, flexibly, and accurately solve addition and subtraction word problems, and add and

subtract within 10.

Priority M.K.R.OA.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

M.K.R.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Priority M.K.R.OA.5

Efficiently, flexibly, and accurately add and subtract within 5.

Work with numbers 11–19 to gain foundations for place value.

Priority M.K.R.NBT.1

Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Spatial Reasoning

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

M.K.SR.G.1 Describe objects in the student environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

M.K.SR.G.2 Correctly name shapes regardless of their orientations or overall size.

M.K.SR.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

Priority M.K.SR.G.4

Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

M.K.SR.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and/or drawing shapes.

M.K.SR.G.6 Use 2-dimensional shapes to compose a variety of larger shapes.

GRADE 1

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.1.DA.DS.1 Generate questions to investigate situations within the classroom.

Collect and consider data.

Priority M.1.DA.DS.2

Collect and use data to consider and decide what data will answer the investigative question. Organize data with drawings, tally marks, or other visual representations.

Analyze the data.

Priority M.1.DA.DS.3

Analyze data sets with up to three categories by making comparisons and/or identifying patterns in the data.

Interpret results.

M.1.DA.DS.4 Interpret and communicate results through structured answers with teacher guidance.

Represent and interpret data.

M.1.DA.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less in each category.

Quantity

Represent and solve problems involving addition and subtraction.

Priority M.1.Q.OA.1

Use addition and subtraction within 20 to efficiently, flexibly, and accurately solve real world word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and/or equations with a

symbol for the unknown number to represent the problem.

Priority M.1.Q.OA.2

Efficiently, flexibly, and accurately solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

M.1.Q.OA.3 Apply and extend properties of operations by selecting and demonstrating strategies to add and subtract.

M.1.Q.OA.4 Demonstrate understanding of subtraction as an unknown-addend problem.

Add and subtract within 20.

M.1.Q.OA.5 Apply and extend counting strategies to addition and subtraction (e.g., by counting on 2 to add 2).

M.1.Q.OA.6 Efficiently, Flexibly, and accurately add within 20, f and subtract within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14, decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

M.1.Q.OA.7 Demonstrate understanding of the meaning of the equal sign and determine if equations involving addition and subtraction are true or false.

M.1.Q.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

Measure lengths indirectly and by iterating length units.

M.1.Q.MD.1 Order three objects according to their length; compare the lengths of two objects indirectly by using a third object.

Priority M.1.Q.MD.2

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Tell and write time.

M.1.Q.MD.3 Tell and write time to the hour and half-hour using analog and digital clocks.

Relationships

Extending the counting sequence.

Priority M.1.R.NBT.1

Count to 120, starting at any number less than 120. In this range, recognize and represent numerals and represent a number of objects with a written numeral.

Understand place value.

Priority M.1.R.NBT.2

Understand that the digits of a two-digit number represent amounts of tens and ones.

- **a.** 10 can be thought of as a bundle of ten ones – called a "ten".
- **b.** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- **c.** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Priority M.1.R.NBT.3

Compare two two-digit numbers based on the values of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

Priority M.1.R.NBT.4

Efficiently, flexibly, and accurately add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

M.1.R.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

M.1.R.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Spatial Reasoning

Reason with shapes and their attributes.

M.1.SR.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus nondefining attributes (e.g., color, orientation, overall size) and represent shapes (build and/or draw) to possess defining attributes.

Priority M.1.SR.G.2

Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and create new shapes from the composite shape.

M.1.SR.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

GRADE 2

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.2.DA.DS.1 Generate questions to investigate topics of interest to students within the classroom, school, or community.

Collect and consider data.

M.2.DA.DS.2 Collect and use data to consider and decide what data will answer the investigative question. Organize data with pictographs, line plots and bar graphs with single-unit scales. Recognize that data can vary for a variety of reasons.

Analyze the data.

Priority M.2.DA.DS.3

Analyze data sets with up to four categories by making comparisons, looking for patterns and/or making predictions.

Interpret results.

Priority M.2.DA.DS.4

Interpret and communicate results through structured answers with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question.

Represent and interpret data.

Priority M.2.DA.MD.9

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

Represent and interpret data.

M.2.DA.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve one step put-together, take-apart, and compare problems

using information presented in a bar graph.

Quantity

Understand place value.

M.2.Q.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

M.2.Q.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Priority M.2.Q.NBT.4

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

Priority M.2.Q.NBT.5

Efficiently, flexibly, and accurately add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Priority M.2.Q.NBT.6

Add up to four two-digit numbers using strategies based on place value and properties of operations.

Priority M.2.Q.NBT.7

Efficiently, flexibly, and accurately add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Demonstrate understanding that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

M.2.Q.NBT.8 Use flexible and efficient strategies to mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

M.2.Q.NBT.9 Explain with words, numbers, and /or pictures why addition and subtraction strategies work, using place value and the properties of operations.

Measure and estimate lengths in standard units.

M.2.Q.MD.1 Measure the length of an object by selecting and using appropriate tools.

M.2.Q.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; explain how the two measurements relate to the size of the unit chosen.

M.2.Q.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

M.2.Q.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.

Relate addition and subtraction to length.

Priority M.2.Q.MD.5

Efficiently, flexibly, and accurately use addition and subtraction within 100 to solve real world word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Priority M.2.Q.MD.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

M.2.Q.MD.8 Efficiently, flexibly, and accurately solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

Relationships

Understand place value.

Priority M.2.R.NBT.1

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.

- **a.** 100 can be thought of as a bundle of ten tens - called a "hundred."
- **b.** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Represent and solve problems involving addition and subtraction.

Priority M.2.R.OA.1

Use addition and subtraction within 100 to flexibly, efficiently, and accurately solve one- and two-step real world word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

M.2.R.OA.2 Efficiently, flexibly, and accurately add and subtract within 20 using flexible and efficient strategies.

Work with equal groups of objects to gain foundations for multiplication.

M.2.R.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of

members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Spatial Reasoning

Reason with shapes and their attributes.

M.2.SR.G.1 Identify and draw shapes based on specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Priority M.2.SR.G.2

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

M.2.SR.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Demonstrate that equal shares of identical wholes need not have the same shape.

Work with time and money.

M.2.SR.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

Work with equal groups of objects to gain foundations for multiplication.

M.2.SR.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

GRADE 3

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.3.DA.DS.1 Generate questions to investigate topics of interest to students that can be answered with a variety of data or data sets.

Collect and consider data.

M.3.DA.DS.2 Collect and consider data in a variety of ways including surveys, groupings, measurement, etc., and ask in what ways the data can be collected to capture as much information as necessary to inform the investigative question.

Analyze the data.

Priority M.3.DA.DS.3

Represent data in a variety of ways including technology. Critically analyze data visualizations, including bar graphs, line plots, and scaled picture graphs with various scales. Analyze data sets with several categories by making comparisons, looking for patterns and/or making predictions and recognize the source and amount of data collected may impact the accuracy.

Interpret results.

Priority M.3.DA.DS.4

Interpret and communicate results, describing difference between groups, with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question.

Solve problems involving measurement and estimation.

M.3.DA.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (gg), kilograms (kgkg), and liters (ll). Add, subtract, multiply, or divide to Efficiently, flexibly, and accurately solve one-step real world word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Represent data.

M.3.DA.MD.3 Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

M.3.DA.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units according to the data— whole numbers, halves, or quarters.

Quantity

Represent and solve problems involving multiplication and division.

M.3.Q.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

M.3.Q.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

Priority M.3.Q.OA.3

Use multiplication and division within 100 to Efficiently, flexibly, and accurately solve real world word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

M.3.Q.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Explore and use properties of multiplication to understand the relationship between multiplication and division.

M.3.Q.OA.5 Use strategies to multiply and divide by applying and extending understanding of the properties of operations.

M.3.Q.OA.6 Demonstrate understanding of division as an unknown-factor problem.

Multiply and divide within 100.

Priority M.3.Q.OA.7

Efficiently, flexibly, and accurately multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.

Solve problems involving the four operations and identify and explain patterns in arithmetic.

Priority M.3.Q.OA.8

Efficiently, flexibly, and accurately solve two-step real world word problems using the four

operations. Represent these problems using visual models and equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental and estimation strategies.

M.3.Q.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

M.3.Q.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

M.3.Q.NBT.2 Flexibly, accurately, and efficiently add and subtract within 1000 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

M.3.Q.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Relationships

Develop understanding of fractions as numbers.

Priority M.3.R.NF.1

Understand a unit fraction as the quantity formed when a whole is partitioned into equal parts and explain that a unit fraction is one of those parts; understand fractions are quantities composed of unit fractions.

Priority M.3.R.NF.2

Understand a fraction as a number; and represent fractions on a number line diagram.

- **a.** Represent one part of a whole divided into equal pieces on a number line by using the segment from zero to one as the whole and dividing it into equal parts based on how many pieces the whole is split into. Understand that each part has the size of one divided by the total number of parts, and that the endpoint of the first part starting at zero marks the location of this fraction on the number line.
- **b.** Represent a fraction *a/b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

Priority M.3.R.NF.3

Explain equivalence of fractions and compare fractions by reasoning about their size.

- **a.** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- **b.** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- **c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole

numbers.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

M.3.R.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by a certain number of unit squares is said to have an area equal to that number of square units.

M.3.R.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Priority M.3.R.MD.7

Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with one side split into two parts is equal to the sum of the areas of two smaller rectangles. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find the area of figures made up of adjoining rectangles by decomposing them into non-overlapping rectangles and adding the areas, applying this technique to solve real world problems.

Spatial Reasoning

Reason with shapes and their attributes.

M.3.SR.G.1 Demonstrate understanding that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Priority M.3.SR.G.2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Solve problems involving measurement and estimation.

M.3.SR.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Efficiently, flexibly, and accurately solve real world word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Geometric measurement: recognize perimeter.

Priority M.3.SR.MD.8

Efficiently, flexibly, and accurately solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

GRADE 4

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.4.DA.DS.1 Generate questions of interest to the students that elicit data, generate ideas based on the questions, and refine the question as necessary.

Collect and consider data.

M.4.DA.DS.2 Determine strategies for collecting and considering data in a variety of ways including with the use of technology, evaluate whether additional data that should be collected to completely address the investigative question.

Analyze the data.

Priority M.4.DA.DS.3

Critically analyze data visualizations, including tables, bar graphs, line plots, or spreadsheets to support a claim related to the investigative question. Ask whether the data collected sufficiently addresses the investigative question.

Interpret results.

Priority M.4.DA.DS.4

Interpret and communicate results, describing difference between groups, with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question.

Represent and interpret data.

M.4.DA.MD.4 Make a line plot to display a data set of measurements in fractions of a unit. Efficiently, flexibly, and accurately solve problems involving addition and subtraction of fractions by using information presented in line plots.

Quantity

Use the four operations with whole numbers to solve problems.

Priority M.4.Q.OA.2

Multiply or divide to efficiently, flexibly, and accurately solve real world word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Priority M.4.Q.OA.3

Efficiently, flexibly, and accurately solve multistep real world word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using visual models and equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using estimation strategies.

Gain familiarity with factors and multiples.

M.4.Q.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

M.4.Q.OA.5 Generate a number or shape pattern that follows a given rule. Identify and explain apparent features of the pattern that were not explicit in the rule itself. Explain why the numbers will continue to alternate in this way.

Generalize place value understanding for multi-digit whole numbers.

Priority M.4.Q.NBT.2

Read, write, and compare multi-digit whole numbers using base-ten numerals, number names, and expanded form using the meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

M.4.Q.NBT.3 Use place value understanding of multi-digit whole numbers to generate estimates to any place less than or equal to 1,000,000 using a variety of estimation strategies.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Priority M.4.Q.NBT.4

Efficiently, flexibly, and accurately add and subtract multi-digit whole numbers using strategies or algorithms.

Priority M.4.Q.NBT.5

Efficiently, flexibly, and accurately multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Priority M.4.Q.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using multiple strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Relationships

Use the four operations with whole numbers to solve problems.

M.4.R.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent these comparison statements as multiplication equations.

Generalize place value understanding for multi-digit whole numbers.

M.4.R.NBT.1 Understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

Extend understanding of fraction equivalence and ordering.

M.4.R.NF.1 Demonstrate understanding of why a fraction is equivalent to another fraction by using visual fraction models (e.g., tape diagrams and number lines), with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Understand and use general principles to recognize and generate equivalent fractions.

Priority M.4.R.NF.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing them to a benchmark fraction. Understand that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, <, and justify the conclusions, e.g., by using a visual fraction model

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Priority M.4.R.NF.3

Efficiently, flexibly, and accurately compose and decompose fractions with a numerator greater than 1 into unit fractions, including fractions greater than one or mixed numbers, to solve situations in context with addition and subtraction of fractions with like denominators.

- **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

M.4.R.NF.4 Flexibly apply and extend previous understandings of multiplication to multiply a fraction by a whole number using visual models in the context of word problems.

a. Understand a fraction *a/b* as a multiple of 1/*b*. For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).

b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5.

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Understand decimal notation for fractions and compare decimal fractions.

Priority M.4.R.NF.5

Explore and explain using models, words, and/or numbers that a fraction with a denominator of 10 is an equivalent fraction with denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100.

Priority M.4.R.NF.6

Explore and explain decimal notation for fractions with denominators of 10 and 100 using models, words, and/or numbers.

M.4.R.NF.7 Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, <and justify the conclusions by using multiple strategies and/or visual models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Priority M.4.R.MD.2

Use the four operations to efficiently, flexibly, and accurately solve real world word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities

using multiple visual models.

Geometric measurement: understand concepts of angles and measure angles.

M.4.R.MD.5 Demonstrate understanding of angles as geometric figures that are formed wherever two rays share a common endpoint and accurately measure an angle.

- **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
- **b.** An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

M.4.R.MD.7 Demonstrate understanding that when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Efficiently, flexibly, and accurately solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.

Spatial Reasoning

Draw and identify lines and angles and classify shapes by properties of their lines and angles.

M.4.SR.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these features in two-dimensional figures.

M.4.SR.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.

M.4.SR.G.3 Recognize a line of symmetry for a two-dimensional figure as a line separating the figure such that the figure can be folded along the line into identical parts. Identify line-symmetric figures and draw lines of symmetry.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

M.4.SR.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec., and express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

M.4.SR.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Geometric measurement: understand concepts of angles and measure angles.

M.4.SR.MD.6 Measure angles in whole-number degrees using a protractor. Represent angles of specified measure.

GRADE 5

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.5.DA.DS.1 Generate questions of interest to the students that elicit data, generate ideas based on the questions, and refine the question as necessary. Pose statistical questions that can compare two variables withing a group, setting, or situation.

Collect and consider data.

M.5.DA.DS.2 Determine strategies for collecting and considering data in a variety of ways including with the use of technology. Understand that data may contain errors (missing values, etc.) and decisions have to be made on how to account for or resolve these issues.

Analyze the data.

Priority M.5.DA.DS.3

Critically analyze data visualizations, including tables, bar graphs, line plots, or spreadsheets to support a claim related to the investigative question. Compare and contrast different data visualizations to determine which transparently communicate results and interpretations.

Interpret results.

Priority M.5.DA.DS.4

Interpret and communicate results, describing difference between groups, with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question. Describe the difference between two groups with different conditions.

Represent data.

M.5.DA.MD.2 Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions to solve problems involving information presented in line plots.

Analyze patterns and relationships.

M.5.DA.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. form ordered pairs consisting of corresponding terms

from the two patterns, and graph the ordered pairs on a coordinate plane.

Quantity

Use equivalent fractions as a strategy to add and subtract fractions.

Priority M.5.Q.NF.1

Add and subtract fractions with unlike denominators (including mixed numbers) using flexible and efficient strategies, including replacing given fractions with equivalent fractions with like denominators. Justify using visual models (e.g., tape diagrams or number lines).

Priority M.5.Q.NF.2

Efficiently, flexibly, and accurately solve real world word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Priority M.5.Q.NF.3

Interpret a fraction as division, where a quantity (the numerator) is divided into equal parts (the denominator). Flexibly and efficiently solve real world word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Assess the reasonableness of answers using estimation strategies.

Priority M.5.Q.NF.4

Apply and extend previous understandings of multiplication to Efficiently, flexibly, and accurately multiply a fraction or whole number by a fraction.

- **a.** Interpret the product *(a/b) x q* as a parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b*. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15.
- **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Priority M.5.Q.NF.5

Interpret multiplication as scaling (resizing) by estimating whether a product will be larger or smaller than a given factor based on the size of the other factor, without performing the indicated multiplication.

- **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product

greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

M.5.Q.NF.6 Flexibly and efficiently solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using fraction models or equations to represent the problem. Assess the reasonableness of answers using mental and estimation strategies.

Priority M.5.Q.NF.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions using fraction models and equations to represent the problem.

- **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3)÷ 4 = 1/12 because (1/12) x 4 = 1/3.
- **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.
- **c.** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Understand the place value system.

Priority M.5.Q.NBT.3

Read, write, and compare decimals to thousandths.

- **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
- **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

M.5.Q.NBT.5 Flexibly, efficiently, and accurately multiply multi-digit whole numbers using strategies or algorithms.

Relationships

Write and interpret numerical expressions.

M.5.R.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

M.5.R.OA.2 Write expressions that record calculations with numbers and interpret numerical expressions without evaluating them . For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Understand the place value system.

M.5.R.NBT.1 Understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and of what it represents in the place to its left.

M.5.R.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.

M.5.R.NBT.4 Use place value understanding of decimals to generate estimates to any place using a variety of estimation strategies.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Priority M.5.R.NBT.6

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using strategies based on place value, properties of operations, and connected to the relationship between multiplication and division including rectangular arrays, partial quotients, and/or area models.

Priority M.5.R.NBT.7

Efficiently, flexibly, and accurately add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Spatial Reasoning

Graph points on the coordinate plane to solve real-world and mathematical problems.

M.5.SR.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one

axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x -coordinate, y-axis and y-coordinate).

Priority M.5.SR.G.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

M.5.SR.G.3 Demonstrate understanding that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

M.5.SR.G.4 Classify two-dimensional figures into categories based on properties.

Convert like measurement units within a given measurement system.

M.5.SR.MD.1 Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real world problems. Assess the reasonableness of answers using mental and estimation strategies.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

M.5.SR.MD.3 Identify volume as an attribute of solid figures and understand concepts of volume measurement.

M.5.SR.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Priority M.5.SR.MD.5

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
- **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

GRADE 6

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.6.DA.DS.1 Formulate and recognize statistical investigative questions that are of interest to students to collect data from online sources and websites, smartphones, sensors, publicly available government agencies, and other modern devices.

Collect and consider data.

Priority M.6.DA.DS.2

Collect and record data with technology to identify and describe the characteristics of data sets. Understand that data can be collected (primary data) or existing data can be obtained from other sources (secondary data).

Analyze the data.

Priority M.6.DA.DS.3

Analyze data visualizations and describe measures of center and variability of quantitative data using appropriate displays (dot plots, boxplots). Describe key features of distributions for the variables including center, variability, and shape.

Interpret results.

M.6.DA.DS.4 Use statistical evidence from analyses to answer the statistical investigative question and communicate results with comprehensive answers with some teacher guidance.

Develop understanding of statistical variability.

M.6.DA.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

M.6.DA.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

M.6.DA.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its

values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

M.6.DA.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Priority M.6.DA.SP.5

Summarize numerical data sets in relation to their context.

- **a.** Reporting data points as the set of a number of observations.
- **b.** Describe what's being measured including how it was measured and its units of measurement.
- **c.** Calculate measures of center (mean and/or median) and variability (interquartile range and/or mean absolute deviation) of the data. Understand the shape of the data and identify any striking deviations (outliers) and connect these features to the context where the data came from.
- **d.** Communicate choice for selecting a given measure of center and variability and the connection to the shape of the data distribution related to the context in which the data were gathered.

Quantity

Compute efficiently, flexibly, and accurately with multi-digit numbers and find common factors and multiples.

M.6.Q.NS.2 Efficiently, flexibly, and accurately divide multi-digit numbers using strategies or algorithms.

M.6.Q.NS.3 Efficiently, flexibly, and accurately add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms for each operation.

Apply and extend previous understandings of numbers to the system of rational numbers.

M.6.Q.NS.5 Explain how positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Priority M.6.Q.NS.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to place any number (integer or rational, positive or negative) on the line (horizontal or vertical), and in the plane.

- **a.** Understand opposite signs of numbers as indicating locations on opposite sides of 0 on the number line, and recognize that, in situations like -(-3), the opposite of the opposite of a number is the number itself (in this example, 3).

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

M.6.Q.NS.7 Understand ordering and absolute value of positive and negative rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

b. Write, interpret, and explain order for rational numbers on a number line in real-world contexts, including the use of inequalities.

c. Interpret absolute value as the distance a number is from zero on a number line, and understand the magnitude of absolute value in real world contexts like comparing temperatures, or understanding the size of a debt.

d. Distinguish comparisons of absolute value from statements about order.

Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

Priority M.6.R.RP.1

Explain the concept of a ratio and efficiently, flexibly, and accurately use ratio language to describe a ratio relationship between two quantities.

M.6.R.RP.2 Understand the concept of a unit rate ab associated with a ratio $a{:}b$ with $b \neq 0$ and use rate language in the context of a ratio relationship.

Priority M.6.R.RP.3

Efficiently, flexibly, and accurately demonstrate understanding of ratio and rate reasoning by solving real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations to find different ways to express the same ratio.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems (like price per item and constant speed)

c. Find percent of a quantity as a rate per 100 (a special ratio out of 100), including finding the whole, a part, and the percent as appropriate for a given context.

d. Use ratios to convert between different measurement units, like inches to feet, considering when it is appropriate to multiply or divide quantities.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Priority M.6.R.NS.1

Interpret and efficiently, flexibly, and accurately determine quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Compute efficiently, flexibly, and accurately with multi-digit numbers and find common factors and multiples.

M.6.R.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understandings of arithmetic to algebraic expressions.

M.6.R.EE.1 Efficiently, flexibly, and accurately write and evaluate numerical expressions involving whole-number exponents.

M.6.R.EE.2 Read, and evaluate expressions efficiently, flexibly, and accurately.

- **a.** Write expressions in which letters stand for numbers to write general instructions like "subtract *y* from 5" as a mathematical expression (5 - *y*).
- **b.** Break down expressions into smaller parts using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
- **c.** Replace values for the variables (evaluating the expression) and use the Order of Operations where appropriate to solve problems including using real-world formulas, and formulas with whole-number exponents.

Priority M.6.R.EE.3

Apply the properties of operations (including the distributive property) efficiently, flexibly, and accurately to generate equivalent expressions.

M.6.R.EE.4 Identify whether two expressions are equivalent (including expressions with one or more variables) as both expressions will always yield the same outcome for any value of a given variable.

Reason about and solve one-variable equations and inequalities.

M.6.R.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

M.6.R.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

Priority M.6.R.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

M.6.R.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

M.6.R.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation.

Spatial Reasoning

Solve real-world and mathematical problems involving area, surface area, and volume.

M.6.SR.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by efficiently, flexibly, and accurately composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Priority M.6.SR.G.2

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

M.6.SR.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

M.6.SR.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Apply and extend previous understandings of numbers to the system of rational numbers.

Priority M.6.SR.NS.8

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

GRADE 7

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

Priority M.7.DA.DS.1

Pose statistical investigative questions about a broader population using samples taken from the population.

Collect and consider data.

M.7.DA.DS.2 Understand information from a sample is valid only if the sample is representative of that population. Understand data can be used to make comparisons between different groups at one point in time and the same group over time.

Analyze the data.

M.7.DA.DS.3 Identify, determine, and interpret measures of center (mean and median) and measures of variability (range, interquartile range) to answer a statistically investigative question, summarizing the distribution of data using the measures of center and variability. Use reasoning about distributions to compare two groups based on the variables.

Interpret results.

Priority M.7.DA.DS 4

Acknowledge that looking beyond the data is feasible and recognize the uncertainty caused by sample-to-sample variability when making comparisons and/or conclusions from data to answer the investigative question.

Use random sampling to draw inferences about a population.

M.7.DA.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Priority M.7.DA.SP.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

M.7.DA.SP.3 Informally assess the degree of overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

M.7.DA.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Investigate chance processes and develop, use, and evaluate probability models.

Priority M.7.DA.SP.5

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

M.7.DA.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability.

M.7.DA.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

M.7.DA.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- **a.** Understand the probability of a compound event is a fraction of the outcomes of the sample space.
- **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- **c.** Design and use a simulation to generate frequencies for compound events.

Quantity

Apply and extend previous understandings of operations with fractions.

M.7.Q.NS.1 Efficiently, flexibly, and accurately apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- **a.** Describe situations in which opposite quantities combine to make 0.
- **b.** Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative

direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

Priority M.7.Q.NS.2

Efficiently, flexibly, and accurately apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand integers can be divided as long as the divisor isn't zero, resulting in rational numbers.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert rational numbers into decimals using flexible, efficient, and accurate strategies, recognizing that the decimal form either ends in 0s or repeats eventually.

Draw, construct, and describe geometrical figures and describe the relationships between them.

M.7.Q.G.1 Efficiently, flexibly, and accurately solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing, and reproducing a scale drawing at a different scale.

Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

M.7.R.RP.1 Efficiently, flexibly, and accurately compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Priority M.7.R.RP.2

Recognize and represent proportional relationships between quantities.

a. Decide whether the relationship between two quantities is proportional using equivalent ratios in a table, graphing on the coordinate plane to see of the graph is a straight line through origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Write equations for proportional relationships.

d. Analyze graphs to understand what the data points indicate about the real-world situation, focusing on points like (0, 0) and (1, r) where r is the unit rate.

M.7.R.RP.3 Efficiently, flexibly, and accurately use proportional relationships to solve multistep ratio and percent problems.

Use properties of operations to generate equivalent expressions.

M.7.R.EE.1 Efficiently, flexibly, and accurately use properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Priority M.7.R.EE.2

Understand that rewriting an expression in different forms in a problem context can bring awareness to parts of the problem and how the quantities in it are related.

Efficiently, flexibly, and accurately solve real-life and mathematical problems using numerical and algebraic expressions and equations.

M.7.R.EE.3 Efficiently, flexibly, and accurately solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Priority M.7.R.EE.4

Use variables to represent quantities in a real-world or mathematical problem and write equations and inequalities to efficiently, flexibly, and accurately to solve problems by reasoning about the quantities.

- **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Compare solving the same problem algebraically vs. with arithmetic, explaining the steps involved in each approach.
- **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solutions of these inequalities and interpret them in context of the problem.

Spatial Reasoning

Draw, construct, and describe geometrical figures and describe the relationships between them.

M.7.SR.G.2 Draw geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

M.7.SR.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-world and mathematical problems involving area, surface area, and volume.

M.7.SR.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; informally show the relationship between the circumference and area of a circle.

M.7.SR.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Priority M.7.SR.G.6

Efficiently, flexibly, and accurately solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

GRADE 8

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.8.DA.DS.1 Formulate statistical investigative questions to articulate research topics and uncover patterns of association seen in bivariate categorical data, that multiple investigative questions may exist for a research topic and must take into account context.

Collect and consider data.

M.8.DA.DS.2 Understand how to interrogate the data to determine how the data were collected, from whom they were collected, what types of variables are in the data, how the variables were measured, and possible outcomes for the variables.

Analyze the data.

Priority M.8.DA.DS.3

Create data visualizations about a data set. Organize and present the data in appropriate ways, including in tables and scatter plots, and incorporate other relevant information that helps to tell a story and support a claim about the data.

Interpret results.

Priority M.8.DA.DS.4

Generalize beyond the sample providing statistical evidence for the conclusion, being sure to address limitations of the sample, evidenced in the data. Consider the reasonableness of the results.

Investigate patterns of association in bivariate data.

M.8.DA.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

M.8.DA.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight

line, and informally assess the model fit by judging the closeness of the data points to the line.

Priority M.8.DA.SP.3
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

M.8.DA.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Quantity

Know that there are numbers that are not rational and approximate them by rational numbers.

Priority M.8.Q.NS.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers efficiently, flexibly, and accurately show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

M.8.Q.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Work with radicals and integer exponents.

M.8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Priority M.8.Q.EE.2
Use square roots and cube roots where p is a positive rational number. Use square root symbols to represent solutions to equations of the form $x^2 = p$. Evaluate square roots of perfect squares. Use cube root symbols to represent solutions to equations of the form $x^3 = p$ and evaluate cube roots of perfect cubes. Know that $\sqrt{2}$ is irrational.

M.8.Q.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

M.8.Q.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Relationships

Understand the connections between proportional relationships, lines, and linear equations.

M.8.R.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

M.8.R.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

Priority M.8.R.EE.7

Efficiently, flexibly, and accurately solve linear equations in one variable.

- **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- **b.** Solve linear equations with rational number coefficients where solution paths may require using the distributive property and combining like terms.

Priority M.8.R.EE.8

Analyze and efficiently, flexibly, and accurately solve pairs of simultaneous linear equations.

- **a.** Understand the solution to a system of linear equations is the point of intersection, because the intersection is a solution to both equations.
- **b.** Solve systems of linear equations using a variety of strategies (algebraically, graphically, numerically in tables, verbally, etc.)
- **c.** Solve real-world and mathematical problems leading to two linear equations in two variables.

Apply and extend previous understandings of arithmetic to algebraic expressions.

M.8.R.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

M.8.R.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

M.8.R.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

Priority M.8.R.F.4

Construct a function to model a linear relationship between two quantities. Determine the rate of

change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

M.8.R.F.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described informally.

Spatial Reasoning

Understand congruence and similarity using physical models, transparencies, or geometry software.

M.8.SR.G.1 Verify experimentally the properties of rotations, reflections, and translations.

- **a.** Line and segment lengths are preserved in rotations, reflections, and translations.
- **b.** Angles are taken to angles of the same measure.
- **c.** Parallel lines are taken to parallel lines.

M.8.SR.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

M.8.SR.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Priority M.8.SR.G.4

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

M.8.SR.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Understand and apply the Pythagorean Theorem.

M.8.G.B.6 Efficiently, flexibly and accurately explain a proof of the Pythagorean Theorem and its converse.

Priority M.8.SR.G.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

M.8.SR.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving area, surface area, and volume.

M.8.SR.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

INTEGRATED MATH 1

A student's credit 1 and 2 math selection should align with their High School and Beyond Plan and be aligned with course equivalency to Algebra 1 or Geometry, or Integrated Math 1 or 2. OSPI acknowledges credit 1 and 2 equivalencies may be designed to address any combination of standards in this document, or additional Common Core Mathematics Standards not stated here, with increasing complexity and depth in each successive year.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

Priority M.HS.DA.DS.1

Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

M.HS.DA.DS.2 Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

Priority M.HS.DA.DS.3

Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

M.HS.DA.DS.4 Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand the concept of a function and use function notation.

M.HS.DA.FIF.1 Understand that a function has a domain (input, dependent elements) and range (output, independent elements), and assigns to each domain element exactly one range element. If f is a function and x is a value of its domain, then the output of f corresponds to the input x. The graph of f is the graph of the equation $y = f(x)$.

M.HS.DA.FIF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Summarize, represent, and interpret data on a single count or measurement variable.

Priority M.HS.DA.SID.1

Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.HS.DA.SID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Priority M.HS.DA.SID.3

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.HS.DA.SID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.HS.DA.SID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- **b.** [Not addressed in this course.]
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

Priority M.HS.DA.SID.7

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

M.HS.DA.SID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Priority M.HS.DA.SID.9
Distinguish between correlation and causation.

Quantity

Reason quantitatively and use units to solve problems.

Priority M.HS.Q.NQ.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Priority M.HS.Q.NQ.2
Define appropriate quantities for the purpose of descriptive modeling.

Priority M.HS.Q.NQ.3
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context within linear, exponential, and quadratic functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph linear, exponential, and quadratic functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** including intercepts, maxima, and minima.

M.HS.Q.FIF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions could be linear, exponential, or quadratic.

Create equations that describe numbers or relationships.

M.HS.Q.ACED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context within linear, quadratic, and exponential equations.

M.HS.Q.ACED.4 Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations within linear, quadratic, and exponential equations.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

Understand the concept of a function and use function notation.

M.HS.Q.FIF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology, within linear, exponential, and quadratic functions.

Relationships

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For a function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. Functions could be linear, exponential, or quadratic.

M.HS.R.FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions could be linear, exponential, or quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Create equations that describe numbers or relationships.

Priority M.HS.R.ACED.1

Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

Priority M.HS.R.ACED.2

Efficiently, flexibly, and accurately create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes

with labels and scales.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions in terms of context, including:

- **a.** Linear functions grow with equal differences over equal intervals and with exponential functions grow with equal factors over equal intervals.
- **b.** Recognizing constant rates per unit interval relative to another.
- **c.** Recognize contexts of growth or decay by a constant percent rate per unit interval relative to another.

M.HS.R.FLE.2 Efficiently, flexibly, and accurately construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically.

Interpret expressions for functions in terms of the situation they model.

Priority M.HS.R.FLE.5

Interpret the parameters in a linear or exponential function in terms of a context.

Understand solving equations as a process of reasoning and explain the reasoning.

Priority M.HS.R.AREI.1

Given that an original equation has a solution, efficiently, flexibly, and accurately select and use strategies to solve the equation, explaining each step, and construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

MHS.R.AREI.3 Efficiently, flexibly, and accurately solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

M.HS.R.AREI.5 Efficiently, flexibly, and accurately demonstrate that systems of two equations in two variables maintain the same solution set when one equation is replaced by the sum of that equation and a multiple of the other equation.

M.HS.R.AREI.6 Efficiently, flexibly, and accurately solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

M.HS.SR.AREI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, exponential, and quadratic.

M.HS.SR.AREI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.5 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.HS.SR.GGPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.HS.SR.GGPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Experiment with transformations in the plane.

Priority M.HS.SR.GCO.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.HS.SR.GCO.2 Efficiently, flexibly, and accurately represent transformations in the plane, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.HS.SR.GCO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Priority M.HS.SR.GCO.4

Develop definitions of rotations, reflections, and translations in terms of angles, circles,

perpendicular lines, parallel lines, and line segments.

M.HS.SR.GCO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Efficiently, flexibly and accurately specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

Priority M.HS.SR.GCO.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.HS.SR.GCO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.HS.SR.GCO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Make geometric constructions.

M.HS.SR.GCO.12 Make formal geometric constructions with a variety of tools and methods.

M.HS.SR.GCO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

INTEGRATED MATH 2

A student's credit 1 and 2 math selection should align with their High School and Beyond Plan and be aligned with course equivalency to Algebra 1 or Geometry, or Integrated Math 1 or 2. OSPI acknowledges credit 1 and 2 equivalencies may be designed to address any combination of standards in this document, or additional Common Core Mathematics Standards not stated here, with increasing complexity and depth in each successive year.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand independence and conditional probability and use them to interpret data.

Priority M.HS.DA.SCP.1

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

M.HS.DA.SCP.2 Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

M.HS.DA.SCP.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B),}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

M.HS.DA.SCP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Priority M.HS.DA.SCP.5

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events.

M.HS.DA.SCP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

M.HS.DA.SCP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Quantity

Understand similarity in terms of similarity transformations.

M.HS.Q.GSRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Define trigonometric ratios and solve problems involving right triangles.

M.HS.Q.GSRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.HS.Q.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

M.HS.Q.GSRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context within linear, exponential, and quadratic functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite it such as the difference of squares, compound interest, and others.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph linear, exponential, and quadratic functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, and integer constants for exponential growth and decay, rewrite a function in different equivalent forms.

- **a.** Factor quadratic functions to verify and explain various properties on graphs, such as zeros and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions.

Priority M.HS.Q.FIF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions could be linear, exponential, or quadratic.

Priority M.HS.Q.ACED.4

Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations within linear, quadratic, and exponential equations.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k, k\, f(x), f(kx)$, and

$f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology, within linear, exponential, and quadratic functions.

Extend the properties of exponents to rational exponents.

M.HS.Q.NRN.1 Efficiently, flexibly, and accurately explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values using a variety of strategies, allowing for a notation for radicals in terms of rational exponents.

M.HS.Q.NRN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

M.HS.Q.NRN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials, limited to quadratic equations.

Relationships

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For a function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. Functions could be linear, exponential, or quadratic.

Priority M.HS.R.FIF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions could be linear, exponential, or quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Write expressions in equivalent forms to solve problems.

Priority M.HS.R.ASSE.3

Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions (linear, quadratic,

exponential)

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** [Not addressed in this course.]
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Create equations that describe numbers or relationships.

Priority M.HS.R.ACED.1

Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** [Not addressed in this course.]
- **b.** Solve by inspection (e.g., for $x^2 = 49$), taking square roots, and factoring, as appropriate to the initial equation.

Solve systems of equations.

M.HS.R.AREI.7 Efficiently, flexibly, and accurately solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Prove geometric theorems.

M.HS.R.GCO.9 Efficiently, flexibly, and accurately prove theorems about lines and angles: vertical angles, transversals, alternate interior and exterior angles, perpendicular bisectors, etc.

M.HS.R.GCO.10 Efficiently, flexibly, and accurately prove theorems about triangles: interior angles, base angles, segments joining midpoint of two sides, and medians of a triangle.

M.HS.R.GCO.11 Efficiently, flexibly, and accurately prove theorems about parallelograms: congruence of opposite sides and opposite angles, properties of diagonals.

Prove theorems involving similarity.

M.HS.R.GSRT.4 Efficiently, flexibly, and accurately prove theorems about triangles including proportionality, triangle similarity, and the Pythagorean Theorem.

Priority M.HS.R.GSRT.5
Efficiently, flexibly, and accurately use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Understand and apply theorems about circles.
M.HS.R.GC.1 Efficiently, flexibly, and accurately prove that all circles are similar.

Priority M.HS.R.GC.2
Identify and describe relationships among inscribed angles, radii, and chords, including how angles formed inside the circle, the circle's radius, and line segments within the circle are related. Understand special cases including angles formed by diameters and how the circle's edge interacts with its radius.

Find arc lengths and areas of sectors of circles.
M.HS.R.GC.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Spatial Reasoning

Understand and apply theorems about circles.
M.HS.SR.GC.3 Construct the inscribed and circumscribed circles of a triangle and efficiently, flexibly, and accurately prove properties of angles for a quadrilateral inscribed in a circle.

Understand similarity in terms of similarity transformations.
Priority M.HS.SR.GSRT.1
Verify experimentally how lines are affected by the center of dilation and how the scale factor changes line segments.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Priority M.HS.SR.GSRT.2
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Translate between the geometric description and the equation for a conic section.
M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem.

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

Explain volume formulas and use them to solve problems.

M.HS.SR.GGMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Priority M.HS.SR.GGMD.3

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations.

Priority M.HS.SR.GMG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Priority M.HS.SR.GMG.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Priority M.HS.SR.GMG.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

INTEGRATED MATH 3

A student's third credit of math should be based on the student's interest and their High School and Beyond Plan. Third credit math courses address math standards with increased complexity and depth from previous math courses. OSPI acknowledges third credit math courses may be designed to address any combination of the standards in this document.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand and evaluate random processes underlying statistical experiments.

M.HS.DA.SIC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

M.HS.DA.SIC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Priority M.HS.DA.SIC.3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Priority M.HS.DA.SIC.4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

M.HS.DA.SIC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Priority M.HS.DA.SIC.6

Evaluate reports based on data.

Summarize, represent, and interpret data on a single count or measurement variable.

M.HS.DA.SID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Priority M.HS.DA.SID.6

Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Models may include any family of functions.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Quantity

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context in any family of functions.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure of an expression to identify ways to rewrite it.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

M.HS.Q.FBF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

M.HS.Q.ASSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.

Analyze functions using different representations.

Priority M.HS.Q.FIF.7

Graph families of functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

a. including intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. [Not addressed in this course.]
e. Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Factor quadratic functions and completing the square to show zeros, extreme values and symmetry in terms of a context.
b. Use properties of exponents to interpret expressions for exponential functions, including non-integer constants for time when applicable in contexts of exponential growth or decay.

Priority M.HS.Q.FIF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology.

Create equations that describe numbers or relationships.

Priority M.HS.Q.ACED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. May include any families of functions.

Priority M.HS.Q.ACED.4

Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. May include any families of functions.

Perform arithmetic operations with complex numbers.

M.HS.Q.NCN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

M.HS.Q.NCN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

M.HS.Q.NCN.7 Solve quadratic equations with real coefficients that have complex solutions.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

M.HS.Q.AAPR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

M.HS.Q.AAPR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

M.HS.Q.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.

M.HS.Q.AAPR.6 Efficiently, flexibly, and accurately rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x) + r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Relationships

Create equations that describe numbers or relationships.

M.HS.R.ACED.1 Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Write expressions in equivalent forms to solve problems.

M.HS.R.ASSE.3 Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For any function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. May include any families of functions, and relative maximums and minimums, end behavior, and periodicity.

Priority M.HS.R.FIF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a non-linear function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Build new functions from existing functions.

M.HS.R.FBF.4 Find inverse functions through focus on relationships between inputs and outputs.

- **a.** Solve and equation of the form $f(x) = c$ that has an inverse and write an equation for the inverse.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Extend the domain of trigonometric functions using the unit circle.

M.HS.SR.FTF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

M.HS.SR.FTF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

Priority M.HS.SR.FTF.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Prove and apply trigonometric identities.

M.HS.SR.FTF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$, given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

M.HS.SR.GGPE.2 Derive the equation of a parabola given a focus and directrix.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

GEOMETRY

A student's credit 1 and 2 math selection should align with their High School and Beyond Plan and be aligned with course equivalency to Algebra 1 or Geometry, or Integrated Math 1 or 2.
OSPI acknowledges credit 1 and 2 equivalencies may be designed to address any combination of standards in this document, or additional Common Core Mathematics Standards not stated here, with increasing complexity and depth in each successive year.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand independence and conditional probability and use them to interpret data.

Priority M.HS.DA.SCP.1

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

M.HS.DA.SCP.2 Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

M.HS.DA.SCP.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B),}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

M.HS.DA.SCP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Priority M.HS.DA.SCP.5

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events.

M.HS.DA.SCP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

M.HS.DA.SCP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Quantity

Understand similarity in terms of similarity transformations.

M.HS.Q.GSRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Define trigonometric ratios and solve problems involving right triangles.

M.HS.Q.GSRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.HS.Q.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

M.HS.Q.GSRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in

applied problems.

Relationships

Solve real-world and mathematical problems involving area, surface area, and volume.

M.HS.R.GCO.9 Efficiently, flexibly, and accurately prove theorems about lines and angles: vertical angles, transversals, alternate interior and exterior angles, perpendicular bisectors, etc.

M.HS.R.GCO.10 Efficiently, flexibly, and accurately prove theorems about triangles: interior angles, base angles, segments joining midpoint of two sides, and medians of a triangle.

M.HS.R.GCO.11 Efficiently, flexibly, and accurately prove theorems about parallelograms: congruence of opposite sides and opposite angles, properties of diagonals.

Prove theorems involving similarity.

M.HS.R.GSRT.4 Efficiently, flexibly, and accurately prove theorems about triangles including proportionality, triangle similarity, and the Pythagorean Theorem.

Priority M.HS.R.GSRT.5

Efficiently, flexibly, and accurately use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Understand and apply theorems about circles.

M.HS.R.GC.1 Efficiently, flexibly, and accurately prove that all circles are similar.

Priority M.HS.R.GC.2

Identify and describe relationships among inscribed angles, radii, and chords, including how angles formed inside the circle, the circle's radius, and line segments within the circle are related. Understand special cases including angles formed by diameters and how the circle's edge interacts with its radius.

Find arc lengths and areas of sectors of circles.

M.HS.R.GC.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Spatial Reasoning

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.5 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.HS.SR.GGPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.HS.SR.GGPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Experiment with transformations in the plane.

Priority M.HS.SR.GCO.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.HS.SR.GCO.2 Efficiently, flexibly, and accurately represent transformations in the plane, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.HS.SR.GCO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Priority M.HS.SR.GCO.4

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

M.HS.SR.GCO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Efficiently, flexibly, and accurately specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

Priority M.HS.SR.GCO.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.HS.SR.GCO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.HS.SR.GCO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Make geometric constructions.

M.HS.SR.GCO.12 Make formal geometric constructions with a variety of tools and methods.

M.HS.SR.GCO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a

circle.

Understand and apply theorems about circles.

M.HS.SR.GC.3 Construct the inscribed and circumscribed circles of a triangle, and efficiently, flexibly, and accurately prove properties of angles for a quadrilateral inscribed in a circle.

Understand similarity in terms of similarity transformations.

Priority M.HS.SR.GSRT.1

Verify experimentally how lines are affected by the center of dilation and how the scale factor changes line segments.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Priority M.HS.SR.GSRT.2

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Determine the equation of a circle of given center and radius using the Pythagorean Theorem.

Explain volume formulas and use them to solve problems.

M.HS.SR.GGMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Priority M.HS.SR.GGMD.3

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations.

Priority M.HS.SR.GMG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Priority M.HS.SR.GMG.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Priority M.HS.SR.GMG.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

ALGEBRA 1

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6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

Priority M.HS.DA.DS.1

Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

M.HS.DA.DS.2 Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

Priority M.HS.DA.DS.3

Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

M.HS.DA.DS.4 Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation

Understand the concept of a function and use function notation.

M.HS.DA.FIF.1 Understand that a function has a domain (input, dependent elements) and range (output, independent elements), and assigns to each domain element exactly one range element. If f is a function and x is a value of its domain, then the output of f corresponds to the input x. The graph of f is the graph of the equation $y = f(x)$.

M.HS.DA.FIF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

Summarize, represent, and interpret data on a single count or measurement variable.

Priority M.HS.DA.SID.1

Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.HS.DA.SID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Priority M.HS.DA.SID.3

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.HS.DA.SID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.HS.DA.SID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- **b.** [Not addressed in this course.]
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

Priority M.HS.DA.SID.7

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

M.HS.DA.SID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Priority M.HS.DA.SID.9
Distinguish between correlation and causation.

Quantity

Reason quantitatively and use units to solve problems.

Priority M.HS.Q.NQ.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Priority M.HS.Q.NQ.2
Define appropriate quantities for the purpose of descriptive modeling.

Priority M.HS.Q.NQ.3
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context within linear, exponential, and quadratic functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite it including the difference of squares, compound interest and others.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph linear, exponential, and quadratic functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Factor quadratic functions to verify and explain various properties on graphs, such as zeros and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions.

M.HS.Q.FIF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions could be linear, exponential, or quadratic.

Create equations that describe numbers or relationships.

M.HS.Q.ACED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context within linear, quadratic, and exponential equations.

M.HS.Q.ACED.4 Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations within linear, quadratic, and exponential equations.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

Understand the concept of a function and use function notation.

M.HS.Q.FIF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology, within linear, exponential, and quadratic functions.

Extend the properties of exponents to rational exponents.

M.HS.Q.NRN.1 Efficiently, flexibly, and accurately explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values using a variety of strategies, allowing for a notation for radicals in terms of rational exponents.

M.HS.Q.NRN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

M.HS.Q.NRN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of

addition, subtraction, and multiplication; add, subtract, and multiply polynomials., limited to quadratic equations.

Relationships

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For a function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. Functions could be linear, exponential, or quadratic.

M.HS.R.FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions could be linear, exponential, or quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Write expressions in equivalent forms to solve problems.

Priority M.HS.R.ASSE.3

Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions (linear, quadratic, exponential)

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** [Not addressed in this course.]
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Create equations that describe numbers or relationships.

Priority M.HS.R.ACED.1

Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

Priority M.HS.R.ACED.2

Efficiently, flexibly, and accurately create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions in terms of context, including:

a. Linear functions grow with equal differences over equal intervals and with exponential functions grow with equal factors over equal intervals.
b. Recognizing constant rates per unit interval relative to another.
c. Recognize contexts of growth or decay by a constant percent rate per unit interval relative to another.

M.HS.R.FLE.2 Efficiently, flexibly, and accurately construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly and quadratically.

Interpret expressions for functions in terms of the situation they model.

Priority M.HS.R.FLE.5

Interpret the parameters in a linear or exponential function in terms of a context.

Understand solving equations as a process of reasoning and explain the reasoning.

Priority M.HS.R.AREI.1

Given that an original equation has a solution, efficiently, flexibly, and accurately select and use strategies to solve the equation, explaining each step, and construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

M.HS.R.AREI.3 Efficiently, flexibly, and accurately solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

a. [Not addressed in this course.]
b. Solve by inspection (e.g., for $x^2 = 49$), taking square roots, and factoring, as appropriate to the initial equation.

Solve systems of equations.

M.HS.R.AREI.5 Efficiently, flexibly, and accurately demonstrate that systems of two equations in two variables maintain the same solution set when one equation is replaced by the sum of that equation and a multiple of the other equation.

M.HS.R.AREI.6 Efficiently, flexibly, and accurately solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

M.HS.R.AREI.7 Efficiently, flexibly, and accurately solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

M.HS.SR.AREI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, exponential, and quadratic.

M.HS.SR.AREI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

ALGEBRA 2

A student's third credit of math should be based on the student's interest and their High School and Beyond Plan. Third credit math courses address math standards with increased complexity and depth from previous math courses. OSPI acknowledges third credit math courses may be designed to address any combination of the standards in this document.

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4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand and evaluate random processes underlying statistical experiments.

M.HS.DA.SIC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

M.HS.DA.SIC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Priority M.HS.DA.SIC.3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Priority M.HS.DA.SIC.4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

M.HS.DA.SIC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Priority M.HS.DA.SIC.6

Evaluate reports based on data.

Summarize, represent, and interpret data on a single count or measurement variable.

M.HS.DA.SID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Priority M.HS.DA.SID.6

Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Models may include any family of functions.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Quantity

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context in any family of functions.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

M.HS.Q.FBF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model linear and exponential situations, and translate between the two forms.

Write expressions in equivalent forms to solve problems.

M.HS.Q.ASSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.

Analyze functions using different representations.

Priority M.HS.Q.FIF.7

Graph families of functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

a. Including intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. [Not addressed in this course.]
e. Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Factor quadratic functions and completing the square to show zeros, extreme values and symmetry in terms of a context.
b. Use properties of exponents to interpret expressions for exponential functions, including non-integer constants for time when applicable in contexts of exponential growth or decay.

Priority M.HS.Q.FIF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions can include polynomial, radical, rational,

logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology.

Create equations that describe numbers or relationships.

Priority M.HS.Q.ACED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. May include any families of functions.

Priority M.HS.Q.ACED.4

Efficiently, flexibly, and accurately rearrange formulas with multiple quantities of interests to highlight and contextualize a specific quantity, using the same reasoning as in solving equations. May include any families of functions.

Perform arithmetic operations with complex numbers.

M.HS.Q.NCN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

M.HS.Q.NCN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

M.HS.Q.NCN.7 Solve quadratic equations with real coefficients that have complex solutions.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

M.HS.Q.AAPR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

M.HS.Q.AAPR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial

M.HS.Q.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.

M.HS.Q.AAPR.6 Efficiently, flexibly, and accurately rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x)+r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Relationships

Create equations that describe numbers or relationships.

M.HS.R.ACED.1 Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Write expressions in equivalent forms to solve problems.

M.HS.R.ASSE.3 Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For any function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Functions can include polynomial,

radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Priority M.HS.R.FIF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a non-linear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Build new functions from existing functions.

M.HS.R.FBF.4 Find inverse functions through focus on relationships between inputs and outputs.

- **a.** Solve and equation of the form *f(x) = c* that has an inverse and write an equation for the inverse.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Extend the domain of trigonometric functions using the unit circle.

M.HS.SR.FTF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle

M.HS.SR.FTF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Priority M.HS.SR.FTF.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

M.HS.SR.FTF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$, given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

Expressing geometric properties with equations.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem, and complete the square to determine the center and radius of a circle from the equation.

M.HS.SR.GGPE.2 Derive the equation of a parabola given a focus and directrix.

HIGH SCHOOL CREDIT 3

A student's third credit of math should be based on the student's interest and their High School and Beyond Plan. Third credit math courses address math standards with increased complexity and depth from previous math courses. OSPI acknowledges third credit math courses may be designed to address any combination of the standards in this document.

Note: Plus standards **(+)** are addressed in credits 3 and 4 mathematics in courses aligned with a student's High School and Beyond Plan.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

M.HS.DA.DS.2 Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

M.HS.DA.DS.4 Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation

Understand the concept of a function and use function notation.

M.HS.DA.FIF.1 Understand that a function has a domain (input, dependent elements) and range (output, independent elements), and assigns to each domain element exactly one range element. If f is a function and x is a value of its domain, then the output of f corresponds to the input x. The graph of f is the graph of the equation $y = f(x)$.

M.HS.DA.FIF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Summarize, represent, and interpret data on a single count or measurement variable.

M.HS.DA.SID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.HS.DA.SID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

M.HS.DA.SID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

M.HS.DA.SID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.HS.DA.SID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.HS.DA.SID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Models may include any family of functions.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

M.HS.DA.SID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

M.HS.DA.SID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

M.HS.DA.SID.9 Distinguish between correlation and causation.

Understand and evaluate random processes underlying statistical experiments.

M.HS.DA.SIC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

M.HS.DA.SIC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

M.HS.DA.SIC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

M.HS.DA.SIC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

M.HS.DA.SIC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant

M.HS.DA.SIC.6 Evaluate reports based on data

Understand independence and conditional probability and use them to interpret data.

M.HS.DA.SCP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

M.HS.DA.SCP.2 Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

M.HS.DA.SCP.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B),}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

M.HS.DA.SCP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

M.HS.DA.SCP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events.

M.HS.DA.SCP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

M.HS.DA.SCP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

M.HS.DA.SCP.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

M.HS.DA.SCP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Calculate expected values and use them to solve problems.

M.HS.DA.SMD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

M.HS.DA.SMD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

M.HS.DA.SMD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

M.HS.DA.SMD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

Use probability to evaluate outcomes of decisions.

M.HS.DA.SMD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

- **a.** Find the expected payoff for a game of chance.
- **b.** Evaluate and compare strategies on the basis of expected values.

M.HS.DA.SMD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

M.HS.DA.SMD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Quantity

Reason quantitatively and use units to solve problems.

M.HS.Q.NQ.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

M.HS.Q.NQ.2 Define appropriate quantities for the purpose of descriptive modeling.

M.HS.Q.NQ.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context in any family of functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph families of functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.
- **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **d.** Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **e.** **(+)** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Factor quadratic functions and completing the square to show zeros, extreme values and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions, including non-integer constants for time when applicable in contexts of exponential growth or decay.

M.HS.Q.FIF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Create equations that describe numbers or relationships.

M.HS.Q.ACED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. May include any families of functions.

M.HS.Q.ACED.4 Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of

interest, using the same reasoning as in solving equations. May include any families of functions.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.
- **c.** **(+)** Compose functions.

M.HS.Q.FBF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model linear and exponential situations, and translate between the two forms.

Understand the concept of a function and use function notation.

M.HS.Q.FIF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Write expressions in equivalent forms to solve problems.

M.HS.Q.ASSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology.

Extend the properties of exponents to rational exponents.

M.HS.Q.NRN.1 Efficiently, flexibly, and accurately explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values using a variety of strategies, allowing for a notation for radicals in terms of rational exponents.

M.HS.Q.NRN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

M.HS.Q.NRN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations with complex numbers.

M.HS.Q.NCN.1 Know there is a complex number i such that i2 = -1, and every complex number has the form a + bi with a and b real.

M.HS.Q.NCN.2 Use the relation i2 = -1 and the commutative, associative, and distributive

properties to add, subtract, and multiply complex numbers.

M.HS.Q.NCN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

M.HS.Q.NCN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

M.HS.Q.NCN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

M.HS.Q.NCN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

M.HS.Q.NCN.7 Solve quadratic equations with real coefficients that have complex solutions.

M.HS.Q.NCN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

M.HS.Q.NCN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Represent and model with vector quantities.

M.HS.Q.NVM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v, |v|, ||v||, \mathbf{v}$).

M.HS.Q.NVM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

M.HS.Q.NVM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

M.HS.Q.NVM.4 (+) Add and subtract vectors.

- **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- **c.** Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector

subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

M.HS.Q.NVM.5 (+) Multiply a vector by a scalar.

- **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.
- **b.** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Perform operations on matrices and use matrices in applications.

M.HS.Q.NVM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

M.HS.Q.NVM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

M.HS.Q.NVM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

M.HS.Q.NVM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

M.HS.Q.NVM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

M.HS.Q.NVM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

M.HS.Q.NVM.12 (+) Work with 2 x 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Solve systems of equations.

M.HS.Q.AREI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

M.HS.Q.AREI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

M.HS.Q.AAPR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a,

the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

M.HS.Q.AAPR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

M.HS.Q.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.

M.HS.Q.AAPR.5 (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

M.HS.Q.AAPR.6 Efficiently, flexibly, and accurately rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x)+r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

M.HS.Q.AAPR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Understand similarity in terms of similarity transformations.

M.HS.Q.GSRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Define trigonometric ratios and solve problems involving right triangles.

M.HS.Q.GSRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.HS.Q.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

M.HS.Q.GSRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Relationships

Interpret functions that arise in applications in terms of the context.

M.HS.R.FIF.4 For any function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. May include any families of functions, and relative maximums and minimums, end behavior, and periodicity.

M.HS.R.FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a non-linear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Create equations that describe numbers or relationships.

M.HS.R.ACED.1 Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Write expressions in equivalent forms to solve problems.

M.HS.R.ASSE.3 Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions.

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions in terms of context, including:

- **a.** Linear functions grow with equal differences over equal intervals and with exponential functions grow with equal factors over equal intervals.
- **b.** Recognizing constant rates per unit interval relative to another.
- **c.** Recognize contexts of growth or decay by a constant percent rate per unit interval relative to another.

M.HS.R.FLE.2 Efficiently, flexibly, and accurately construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly and quadratically, or as a polynomial function.

Interpret expressions for functions in terms of the situation they model.

M.HS.R.FLE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.1 Given that an original equation has a solution, efficiently, flexibly, and accurately select and use strategies to solve the equation, explaining each step, and construct a viable argument to justify a solution method.

M.HS.R.AREI.3 Efficiently, flexibly, and accurately solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

M.HS.R.AREI.5 Efficiently, flexibly, and accurately demonstrate that systems of two equations in two variables maintain the same solution set when one equation is replaced by the sum of that equation and a multiple of the other equation.

M.HS.R.AREI.6 Efficiently, flexibly, and accurately solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Solve systems of equations.

M.HS.R.AREI.7 Efficiently, flexibly, and accurately solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Build new functions from existing functions.

M.HS.R.FBF.4 Find inverse functions through focus on relationships between inputs and outputs.

- **a.** Solve an equation of the form f(x) = c that has an inverse and write an equation for the inverse.
- **b.** **(+)** Verify by composition that one function is the inverse of another.
- **c.** **(+)** Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. **(+)** Produce an invertible function from a non-invertible function by restricting the domain.

M.HS.R.FBF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Prove geometric theorems.

M.HS.R.GCO.9 Efficiently, flexibly, and accurately prove theorems about lines and angles: vertical angles, transversals, alternate interior and exterior angles, perpendicular bisectors, etc.

M.HS.R.GCO.10 Efficiently, flexibly, and accurately prove theorems about triangles: interior angles, base angles, segments joining midpoint of two sides, and medians of a triangle.

M.HS.R.GCO.11 Efficiently, flexibly, and accurately prove theorems about parallelograms: congruence of opposite sides and opposite angles, properties of diagonals.

Prove theorems involving similarity.

M.HS.R.GSRT.4 Efficiently, flexibly, and accurately prove theorems about triangles including proportionality, triangle similarity, and the Pythagorean Theorem.

M.HS.R.GSRT.5 Efficiently, flexibly, and accurately use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Apply trigonometry to general triangles.

M.HS.R.GSRT.9 (+) Derive the formula $A = 1/2\ ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

M.HS.R.GSRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

M.HS.R.GSRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Understand and apply theorems about circles.

M.HS.R.GC.1 Efficiently, flexibly, and accurately prove that all circles are similar.

M.HS.R.GC.2 Identify and describe relationships among inscribed angles, radii, and chords, including how angles formed inside the circle, the circle's radius, and line segments within the circle are related. Understand special cases including angles formed by diameters and how the circle's edge interacts with its radius.

M.HS.R.GC.5 Derive using similarity the fact that the length of the arc intercepted by an angle is

proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Spatial Reasoning

Extend the domain of trigonometric functions using the unit circle.

M.HS.SR.FTF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

M.HS.SR.FTF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

M.HS.SR.FTF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

M.HS.SR.FTF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodicity phenomena with trigonometric functions.

M.HS.SR.FTF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

M.HS.SR.FTF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

M.HS.SR.FTF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

M.HS.SR.FTF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$, given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

M.HS.SR.FTF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem, and completing the square to determine the center of a circle from the equation.

M.HS.SR.GGPE.2 Derive the equation of a parabola given a focus and directrix.

M.HS.SR.GGPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact

that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.5 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.HS.SR.GGPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.HS.SR.GGPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Experiment with transformations in the plane.

M.HS.SR.GCO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.HS.SR.GCO.2 Efficiently, flexibly, and accurately represent transformations in the plane, e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.HS.SR.GCO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

M.HS.SR.GCO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

M.HS.SR.GCO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Efficiently, flexibly, and accurately specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

M.HS.SR.GCO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.HS.SR.GCO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.HS.SR.GCO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from

the definition of congruence in terms of rigid motions.

Make geometric constructions.

M.HS.SR.GCO.12 Make formal geometric constructions with a variety of tools and methods.

M.HS.SR.GCO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Understand and apply theorems about circles.

M.HS.SR.GC.3 Construct the inscribed and circumscribed circles of a triangle and efficiently, flexibly, and accurately prove properties of angles for a quadrilateral inscribed in a circle.

M.HS.SR.GC.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Understand similarity in terms of similarity transformations.

M.HS.SR.GSRT.1 Verify experimentally how lines are affected by the center of dilation and how the scale factor changes line segments.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

M.HS.SR.GSRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Explain volume formulas and use them to solve problems.

M.HS.SR.GGMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

M.HS.SR.GGMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

M.HS.SR.GGMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Represent and solve equations and inequalities graphically.

M.HS.SR.AREI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

M.HS.SR.AREI.11 Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

M.HS.SR.AREI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Apply geometric concepts in modeling situations.

M.HS.SR.GMG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

M.HS.SR.GMG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

M.HS.SR.GMG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

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