Washington Office of Superintendent of
PUBLIC INSTRUCTION

Washington State K-12 Learning Standards for Mathematics

WASHINGTON STATE K-12 LEARNING STANDARDS FOR MATHEMATICS

2026

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TABLE OF CONTENTS

K–12 LEARNING STANDARDS FOR MATHEMATICS ADOPTION STATEMENT

I am pleased to present to you the final Washington State K–12 Learning Standards for Mathematics. This set of standards is built upon the foundation of the Common Core State Standards for Mathematics adopted in 2011.

The standards incorporate improvements suggested by Washington educators and industry leaders. With the addition of data science across all grades and an emphasis on flexibility, efficiency, and accuracy in mathematical fluency, the revisions aim to support student learning for 21st century careers. The standards have been prioritized to reflect students' essential learning at each grade level along with supporting standards to help students build on their learning and understanding. The standards represent the input of hundreds of K–12 educators, educational partners, Washington state commissions, and higher education faculty.

These standards embody Washington state's commitment to providing all students with a world-class education that prepares them for postsecondary pathways, careers, and civic engagement. The revised standards for mathematics retains the strengths of the previous standards and include the following improvements:

- Incorporation of data science standards across all grade levels to foster data literacy in early grades and create and critically examine data in later grades.
- Development of student learning to focus on flexible, efficient, and accurate problem-solving strategies.
- Alignment of high school credits in mathematics to support students' High School and Beyond Plans and postsecondary goals.

While revising these standards, the Office of Superintendent of Public Instruction (OSPI) received many valuable comments and suggestions. I want to thank everyone who contributed their feedback, ideas, and determination to make the standards useful for our students, families, educators, and communities. I am proud of the work we did together. Our collaboration will continue as we launch additional supporting documents and technical support for all school districts across the state.

All of Washington state's K–12 learning standards are subject to continuous review and improvement for the benefit of our state's students. We hope you will find that these standards support a strong mathematics program in your school or district.

Adopted on this 18th day of June 2026,

Chris Reykdal
Superintendent of
Public Instruction

INTRODUCTION

Revisions to the Washington State K–12 Learning Standards for Mathematics (WA Math 2026) create opportunities for teaching and learning that honors math as a dynamic way of understanding and explaining the world and daily life. Connecting ideas within and across grades equips students with a deeper understanding of math that prepares them to be flexible problem solvers throughout and beyond their K–12 experience.

The revised standards amplify connections to real-world applications through the integration of data science and mathematics that can connect to student interest and communities. Problem-based learning is more accessible through the restructured standards which move related standards closer together. Related standards then act as a connection to a central idea, offering students and teachers multiple mathematical standards to interact with a core concept.

Embedded in the WA Math 2026 spreadsheet are examples from the revised standards that connect math content standards to the language structures of the WIDA standards. This alignment supports demonstration of students' understanding through both written work and discussion.

Key Goals of the Revisions

Revisions were guided by the following goals:

- **Structure and integrity**—Support student learning progressions and educator access to nationally aligned resources to support high quality math instruction.
- **Data Science**—Ensure students can collect, analyze, understand, and critique data in a technologically data-driven world.
- **Uplift**—Center the Standards for Mathematical Practice to promote multiple ways of thinking and doing math and to help students see its value in their lives.
- **Clarity**—Shift to "flexibly, efficiently, and accurately" to provide clarity in what it means to be mathematically fluent.
- **Determine**—Clearly identify the content included in the first two credits of high school math.
- **Prioritize**—Clearly identify standards to ensure all students have equitable access to the knowledge and skills that can be leveraged, both in school and beyond.

Structure and Integrity of the Standards

The standards provide students with the opportunity to see and explore connections within and across grades through the reorganization of the standards within Data Analysis, Quantities, Relationships, and Spatial Reasoning domains. Within the new structure of four domains, districts can align instructional materials to the Common Core because the root coding of the original standards remains. For example, **5.MD.B.2**, a measurement and data standard about line plots of data sets is now **M.5.DA.MD.2** to reflect its interconnection with other standards within the data analysis (DA) domain.

Inclusion of Data Science Standards

Data science standards have been added to all grades. In a world where industries and communities rely heavily on data, students must be able to interpret, analyze, and make decisions based on reliable information. The data science standards:

- Build from the Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) framework (American Statistical Association).
- Connect math content—algebra, geometry, fractions, measurement, statistics—to real-world inquiries.
- Encourage cross-disciplinary connections (social studies, science, English language arts, John McCoy (lulilaš) Since Time Immemorial).

See the handout *Understanding and Using Data Science* in the Resources section of the Mathematics K–12 Learning Standards webpage.

Uplifting the Standards for Mathematical Practice

The standards seek to recenter the Standards for Mathematical Practice (SMPs) as the foundation for mathematics teaching and learning. The SMPs are the same across all grades. The SMPs are the thinking and decision making of mathematical problem solving. All SMPs can be made clear through teaching practices that facilitate student discourse, collaborative problem solving, and communicating about thinking in reflection, justification, and critique. Teaching and learning using SMPs creates space for dialogue, problem-posing, and the transfer of learning from student to student and moves the SMPs from an individual experience to a shared lens for exploring content standards.

The definitions of the SMPS can be found in Appendix A.

Providing Clarity

The standards clarify and update language around fluency in the standards.

One example is the shift from the term "fluently" to "efficiently, flexibly, and accurately." This means students can use a variety of strategies to solve problems in ways that are effective and lead to correct solutions across different situations. This approach supports strategies that are built across grade levels and encourages flexible mathematical thinking.

Another example is the move away from the term "from memory." In the original Common Core, this phrase was not meant to emphasize speed or rote repetition. The updated language instead supports a focus on understanding. An example of the wording change in this 3rd grade standard:

- Old **3.OA.C.7**: "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers."

- New **M.3.Q.OA.7**: "Efficiently, flexibly, and accurately multiply and divide within 100, using

strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations."

See the handout *Fluency in Mathematics Standards* in the Resources section of the Mathematics K–12 Learning Standards webpage.

Determining Standards at the High School Level

The standards have been revised to clarify the first two years of high school math should include linear, exponential, and quadratic families of functions. Additional functions can be approached in a student's third credit of high school math aligned with their High School and Beyond Plan. This is demonstrated more specifically in Algebra and Functions standards that previously addressed content that pertained to all families of functions.

See the handout *High School Math Standards* in the Resources section of the Mathematics K–12 Learning Standards webpage.

Priority Learning Standards

Priority Learning Standards are the most essential academic skills and concepts students need to succeed from one grade range to the next. These standards were selected to ensure all students have equitable access to the knowledge and skills that can be leveraged, both in school and beyond. Other learning standards serve to support and reinforce the Priority Learning Standards. The Priority Learning Standards identify the universal set of standards that each and every student should have the opportunity to learn and be able to do.

Because state standards encompass many learning goals for each subject and grade, OSPI collaborated with groups of educators from kindergarten through 12th grade over two years to identify the standards that are most critical to focus on in teaching and assessment. The process was guided by national best practices. Educators had multiple opportunities to review and give feedback before the Priority Learning Standards were finalized.

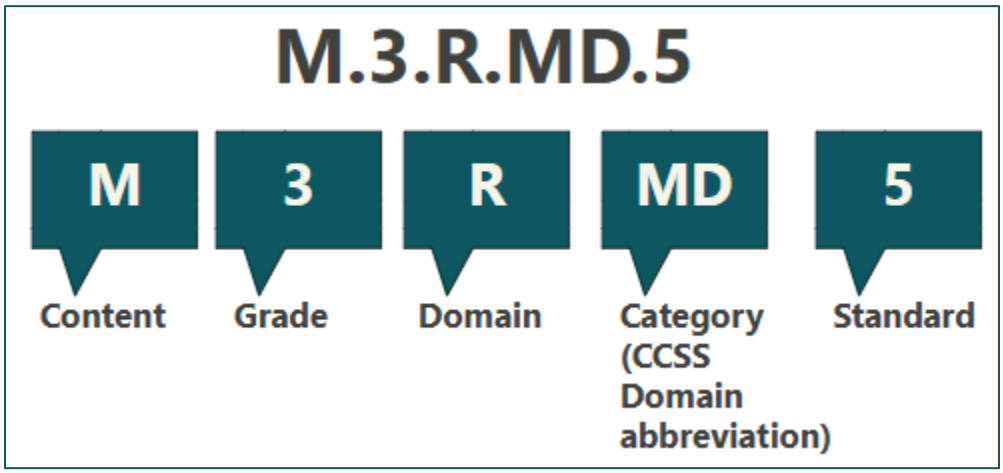
Priority Learning Standards were identified using the following criteria:

- **Endurance**: Will this skill or knowledge still be valuable beyond a single test, school year, or post-high school?
- **Leverage**: Is this skill or knowledge useful across multiple subjects?
- **Readiness**: Does this skill or knowledge prepare students for success in the next grade level, course, or post-high school option?

How to Read the Standards

The Washington State K–12 Learning Standards for Mathematics (WA Math 2026) expands on the structure of the Common Core State Standards for Mathematics. The order of the standards, domains, and categories within a grade level do not indicate the order in which they should be taught.

Figure 1: Math code example



Content code tells which content area or subject the standard is for. The standards use "M".

Grade level or grade band tells which grade level(s) the mathematics standard is for:

- Kindergarten uses "K".
- Grades 1 through 8 use the number for the grade level.
- High school standards use "HS".

Domains are larger groups of standards that highlight their interconnected nature through broad understandings of:

- Data Analysis "DA"
- Quantities "Q"
- Relationships "R"
- Spatial Reasoning "SR"

Categories are groups of standards which align with the original Common Core domains and cluster headings in K–8 (e.g. WA Math 2026 Category of OA comes from the Common Core domain Operations and Algebraic Thinking) and conceptual categories and domains in high school [e.g. WA Math 2026 category of ASSE comes from Common Core HS conceptual category Algebra (A) and Seeing Structure in Expressions domain (SSE)]. For a list of all Common Core domains and their coding, please see the Glossary tab of the 2026 Mathematics Learning Standards Excel file.

For example, 2nd grade standards in Spatial Reasoning are grouped into the following categories:

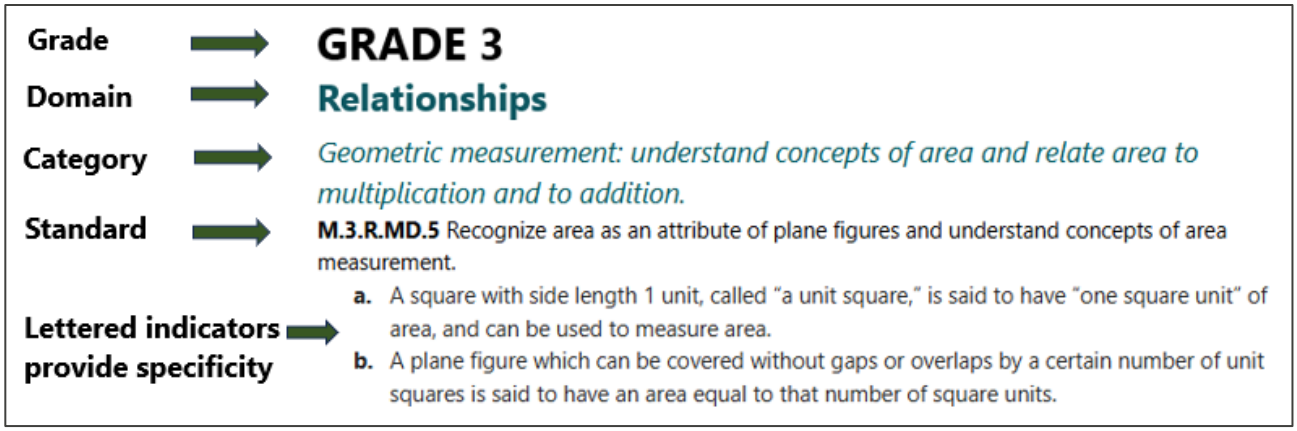
- Geometry (G): Reason with shapes and their attributes.
- Measurement and Data (MD): Work with time and money.
- Operations and Algebraic Thinking (OA): Work with equal groups of objects to gain foundations for multiplication.

Standards define what students should understand and be able to do, and have a number from 1 to 13 in the code.

Lettered indicators provide specificity for the numbered standards. Some standards have these indicators, others do not.

Figure 2 shows an example of a standard with the structure elements labeled.

Figure 2: Example within Word Document



Key Features of the Domains

The Washington State K–12 Learning Standards for Mathematics (WA Math 2026) is organized into four broad domains. This reorganization makes it easier to see connections between mathematical concepts and real-world applications. Each domain is designed to promote critical thinking, problem solving, and meaningful engagement with math. The placement of a standard in a particular domain does not limit its relevance or alignment to other domains.

Data Analysis

Standards that involve formulating statistical investigative questions, collecting, and considering data including measurements, creating data visualizations (ranging from bar graphs and histograms to scatterplots and complex representations as appropriate for the grade), and interpreting results and communicating justification for conclusions.

Quantity

Standards that involve understanding and comparing quantities through mathematical operations. This starts with analysis of magnitude and comparison of quantities in the youngest grades (including math operations and symbols) and mathematical fluency using efficient, flexible, and accurate strategies. This moves towards using math operations for place value, decimal, and fractional reasoning in middle and late elementary school. Quantity culminates with examining number systems (real, integers, rational, irrational, etc.) in middle and high school grades**.**

Relationships

Standards that develop strategic reasoning strategies and understanding of quantities as parts of a relationship, and interpreting and communicating about the relationship in context. Standards in this domain connect to concepts in data analysis, quantity, and spatial reasoning. In elementary students examine relationships through identifying parts and wholes, using mathematical fluency strategies to perform operations with whole numbers, fractions, and decimals, connect decomposing and recomposing number strategies to shapes and fractions, then to geometry principles. Middle school students examine the relationships between parts to expressions and equations, proportionality, and set the groundwork for functions and abstract algebraic relationships in high school. Mathematical modeling of real-world relationships and problem solving are embedded within relational mathematical understanding across all grades.

Spatial Reasoning

Standards that involve understanding shape and spread, both in geometry and data analysis (graphing). Standards in this domain also connect to relationships, quantity, and data analysis. In elementary grades, students define and examine attributes of shapes and geometric principles, and the quantities/measurements associated with those attributes. Middle school students extend their understanding within graphing relationships and geometric principles in the coordinate plane. High school students examine complex attributes of geometry, foundational trigonometric principles, and graphing real-world concepts to examine the meaning of shape and spread of data or a

function in context.

For more information see the handout *Math Domain Revisions* in the Resources section of the Mathematics K–12 Learning Standards webpage.

Table 1: Domain Revisions

CCSS MATH (2011) DOMAINS and CONCEPTUAL CATEGORIES	WA MATH 2026 DOMAINS	WA MATH 2026 CATEGORIES
Broad categories that grouped related mathematical content together, aligned to areas of mathematics as an academic study.	Larger groups of standards that highlight their interconnected nature through broad understandings, aligned to application in real-world contexts.	Groups of standards with codes that align to the original Common Core domains to assist with connections to the original Common Core standards. This will help present reviews and future adoptions of high-quality instructional materials.
Thirty-six domains across K–12 mathematics. 6 – Elementary domains 6 – Middle School domains 24 – High School domains	Four domains that are the same across all grades in K–12 mathematics (see Key Features of Domains).	The root numbering within the mathematics standards (grounded in the original 36 domains and conceptual categories) has been carried forward in WA Math 2026 as categories to maintain alignment of resources for districts.
Number of standards: 342	Number of standards: 381	Priorities: 134
Standards By Grade: K: 22 1: 21 2: 26 3: 25 4: 28 5: 26 6: 29 7: 24 8: 28 HS: 157	Standards By Grade: K: 26 1: 25 2: 30 3: 29 4: 32 5: 30 6: 33 7: 28 8: 32 HS: 160	Priorities By Grade: K: 12 1: 10 2: 13 3: 11 4: 13 5: 13 6: 11 7: 9 8: 10 HS: 32

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.K.DA.DS.1 Generate questions to investigate situations within the classroom.

Collect and consider data.

Priority M.K.DA.DS.2
Collect or consider data through organizing objects or drawing pictures to represent and communicate observations.

Analyze the data.

Priority M.K.DA.DS.3
Analyze data sets by noticing and describing patterns in data-rich situations.

Interpret results.

M.K.DA.DS.4 Interpret and communicate results through structured answers with teacher guidance.

Describe and compare measurable attributes.

M.K.DA.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

M.K.DA.MD.2 Directly compare two objects with a measurable attribute in common to see which object has "more of"/"less of" the attribute and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

M.K.DA.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

Quantity

Know number names and the count sequence.

Priority M.K.Q.CC.1

Count to 100 by ones and by tens.

M.K.Q.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Priority M.K.Q.CC.3

Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

Priority M.K.Q.CC.4

Understand the relationship between numbers and quantities; connect counting to the number of items in the set.

- **a.** When counting objects, use number names in the correct order, matching each number to exactly one object and each object to exactly one number.
- **b.** Understand that the last number name said represents the number of objects counted, and that this total stays the same no matter how the objects are arranged or the order in which they are counted.
- **c.** Understand that each successive number name refers to a quantity that is one larger.

Priority M.K.Q.CC.5

Count to answer "how many?" questions about as many as 20 objects arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers

Priority M.K.Q.CC.6

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

M.K.Q.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Relationships

Represent and solve problems involving addition and subtraction.

M.K.R.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Priority M.K.R.OA.2

Efficiently, flexibly, and accurately solve addition and subtraction word problems, and add and subtract within 10, e.g., using objects or drawings to represent the problem.

Priority M.K.R.OA.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition with a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

M.K.R.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

Priority M.K.R.OA.5

Efficiently, flexibly, and accurately add and subtract within 5.

Work with numbers 11–19 to gain foundations for place value.

Priority M.K.R.NBT.1

Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition with a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Spatial Reasoning

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

M.K.SR.G.1 Describe objects in the student environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

M.K.SR.G.2 Correctly name shapes regardless of their orientations or overall size.

M.K.SR.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

Priority M.K.SR.G.4

Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

M.K.SR.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and/or drawing shapes.

M.K.SR.G.6 Use 2-dimensional shapes to compose a variety of larger shapes. *For example,* "*Can you join these two triangles with full sides touching to make a rectangle?*"

GRADE 1

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.1.DA.DS.1 Generate questions to investigate situations within the classroom.

Collect and consider data.

Priority M.1.DA.DS.2
Collect and use data to consider and decide what data will answer the investigative question. Organize data with drawings, tally marks, or other visual representations.

Analyze the data.

Priority M.1.DA.DS.3
Analyze data sets with up to three categories by making comparisons and/or identifying patterns in the data.

Interpret results.

M.1.DA.DS.4 Interpret and communicate results through structured answers with teacher guidance.

Represent and interpret data

M.1.DA.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less in each category.

Quantity

Represent and solve problems involving addition and subtraction.

Priority M.1.Q.OA.1

Use addition and subtraction within 20 to efficiently, flexibly, and accurately solve real world word problems involving situations of adding to, taking from , putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem.

Priority M.1.Q.OA.2

Efficiently, flexibly, and accurately solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

M.1.Q.OA.3 Apply and extend properties of operations by selecting and demonstrating strategies to add and subtract.

M.1.Q.OA.4 Demonstrate understanding of subtraction as an unknown-addend problem. For example, subtract 10-8 by finding the number that makes 10 when added to 8.

Add and subtract within 20.

M.1.Q.OA.5 Apply and extend counting strategies to addition and subtraction (e.g., by counting on 2 to add 2).

M.1.Q.OA.6 Efficiently, flexibly, and accurately add within 20, and subtract within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

M.1.Q.OA.7 Demonstrate understanding of the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

M.1.Q.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?.*

Measure lengths indirectly and by iterating length units.

M.1.Q.MD.1 Order three objects according to their length; compare the lengths of two objects indirectly by using a third object.

Priority M.1.Q.MD.2

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlap.

Tell and write time.

M.1.Q.MD.3 Tell and write time to the hour and half-hour using analog and digital clocks.

Relationships

Extending the counting sequence.

Priority M.1.R.NBT.1

Count to 120, starting at any number less than 120. In this range, recognize and represent numerals and represent a number of objects with a written numeral.

Understand place value.

Priority M.1.R.NBT.2

Understand that the digits of a two-digit number represent amounts of tens and ones.

- **a.** 10 can be thought of as a bundle of ten ones - called a "ten."
- **b.** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- **c.** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Priority M.1.R.NBT.3

Compare two two-digit numbers based on the values of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

Priority M.1.R.NBT.4

Efficiently, flexibly, and accurately add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

M.1.R.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

M.1.R.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Spatial Reasoning

Reason with shapes and their attributes.

M.1.SR.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus nondefining attributes (e.g., color, orientation, overall size) and represent shapes (build and/or draw) to possess defining attributes.

M.1.SR.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and create new shapes from the composite shape.

Priority M.1.SR.G.3

Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

GRADE 2

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.2.DA.DS.1 Generate questions to investigate topics of interest to students within the classroom, school, or community.

Collect and consider data.

M.2.DA.DS.2 Collect and use data to consider and decide what data will answer the investigative question. Organize data with pictographs, line plots and bar graphs with single-unit scales. Recognize that data can vary for a variety of reasons.

Analyze the data.

Priority M.2.DA.DS.3

Analyze data sets with up to four categories by making comparisons, looking for patterns and/or making predictions.

Interpret results.

Priority M.2.DA.DS.4

Interpret and communicate results through structured answers with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question.

Represent and interpret data.

Priority M.2.DA.MD.9

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

M.2.DA.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve one step put-together, take-apart, and compare problems using information presented in a bar graph.

Quantity

Understand place value.

M.2.Q.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

M.2.Q.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Priority M.2.Q.NBT.4

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

Priority M.2.Q.NBT.5

Efficiently, flexibly, and accurately add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Priority M.2.Q.NBT.6

Add up to four two-digit numbers using strategies based on place value and properties of operations.

Priority M.2.Q.NBT.7

Efficiently, flexibly, and accurately add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Demonstrate understanding that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

M.2.Q.NBT.8 Use flexible and efficient strategies to mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

M.2.Q.NBT.9 Explain with words, numbers, and /or pictures why addition and subtraction strategies work, using place value and the properties of operations.

Measure and estimate lengths in standard units.

M.2.Q.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

M.2.Q.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; explain how the two measurements relate to the size of the unit chosen.

M.2.Q.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

Priority M.2.Q.MD.4

Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.

Relate addition and subtraction to length.

Priority M.2.Q.MD.5

Efficiently, flexibly, and accurately use addition and subtraction within 100 to solve real world word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Priority M.2.Q.MD.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . and represent whole-number sum s and differences within 100 on a number line diagram.

Work with time and money.

M.2.Q.MD.8 Efficiently, flexibly, and accurately solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Relationships

Understand place value.

Priority M.2.R.NBT.1

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- **a.** 100 can be thought of as a bundle of ten tens - called a "hundred."
- **b.** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Represent and solve problems involving addition and subtraction.

Priority M.2.R.OA.1

Use addition and subtraction within 100 to efficiently, flexibly, and accurately solve one- and two-step real world word problems involving situations of adding to, taking from , putting together,

taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

M.2.R.OA.2 Efficiently, flexibly, and accurately add and subtract within 20 using flexible and efficient strategies.

Work with equal groups of objects to gain foundations for multiplication.

M.2.R.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Spatial Reasoning

Reason with shapes and their attributes.

M.2.SR.G.1 Identify and draw shapes based on specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Priority M.2.SR.G.2

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

M.2.SR.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Demonstrate that equal shares of identical wholes need not have the same shape.

Work with time and money.

M.2.SR.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

Work with equal groups of objects to gain foundations for multiplication.

M.2.SR.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

GRADE 3

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.3.DA.DS.1 Generate questions to investigate topics of interest to students that can be answered with a variety of data or data sets.

Collect and consider data.

M.3.DA.DS.2 Collect and consider data in a variety of ways including surveys, groupings, measurement, etc., and ask in what ways the data can be collected to capture as much information as necessary to inform the investigative question.

Analyze the data.

Priority M.3.DA.DS.3

Represent data in a variety of ways including technology. Critically analyze data visualizations, including bar graphs, line plots, and scaled picture graphs with various scales. Analyze data sets with several categories by making comparisons, looking for patterns and/or making predictions and recognize the source and amount of data collected may impact the accuracy.

Interpret results.

Priority M.3.DA.DS.4

Interpret and communicate results, describing difference between groups, with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question.

Solve problems involving measurement and estimation.

M.3.DA.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (*g*), kilograms (*kg*), and liters (*l*). Add, subtract, multiply, or divide to efficiently, flexibly, and accurately solve one-step real world word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Represent data.

M.3.DA.MD.3 Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

M.3.DA.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units according to the data— whole numbers, halves, or quarters.

Quantity

Represent and solve problems involving multiplication and division.

M.3.Q.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

M.3.Q.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

Priority M.3.Q.OA.3
Use multiplication and division within 100 to efficiently, flexibly, and accurately solve real world word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

M.3.Q.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?.*

Explore and use properties of multiplication to understand the relationship between multiplication and division.

M.3.Q.OA.5 Use strategies to multiply and divide by applying and extending understanding of the properties of operations.

M.3.Q.OA.6 Demonstrate understanding of division as an unknown-factor problem.

Multiply and divide within 100.

Priority M.3.Q.OA.7
Efficiently, flexibly, and accurately multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.

Solve problems involving the four operations and identify and explain patterns in arithmetic.

Priority M.3.Q.OA.8

Efficiently, flexibly, and accurately solve two-step real world word problems using the four operations. Represent these problems using visual models and equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental and estimation strategies.

M.3.Q.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. F*or example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Use place value understanding and properties of operations to perform multi-digit arithmetic.

M.3.Q.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

M.3.Q.NBT.2 Efficiently, flexibly, and accurately add and subtract within 1000 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

M.3.Q.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Relationships

Develop understanding of fractions as numbers.

Priority M.3.R.NF.1

Understand a unit fraction as the quantity formed when a whole is partitioned into equal parts and explain that a unit fraction is one of those parts; understand fractions are quantities composed of unit fractions.

Priority M.3.R.NF.2

Understand a fraction as a number; and represent fractions on a number line diagram.

- **a.** Represent one part of a whole divided into equal pieces on a number line by using the segment from zero to one as the whole and dividing it into equal parts based on how many pieces the whole is split into. Understand that each part has the size of one divided by the total number of parts, and that the endpoint of the first part starting at zero marks the location of this fraction on the number line.
- **b.** Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

Priority M.3.R.NF.3

Explain equivalence of fractions and compare fractions by reasoning about their size.

- **a.** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- **b.** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- **c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
- **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

M.3.R.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

- **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
- **b.** A plane figure which can be covered without gaps or overlaps by a certain number of unit squares is said to have an area equal to that number of square units.

M.3.R.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Priority M.3.R.MD.7

Relate area to the operations of multiplication and addition.

- **a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- **b.** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- **c.** Use tiling to show in a concrete case that the area of a rectangle with one side split into two parts is equal to the sum of the areas of two smaller rectangles. Use area models to represent the distributive property in mathematical reasoning.
- **d.** Recognize area as additive. Find the area of figures made up of adjoining rectangles by decomposing them into non-overlapping rectangles and adding the areas, applying this technique to solve real world problems.

Spatial Reasoning

Reason with shapes and their attributes.

M.3.SR.G.1 Demonstrate understanding that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Priority M.3.SR.G.2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Solve problems involving measurement and estimation.

M.3.SR.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Efficiently, flexibly, and accurately solve real world word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Geometric measurement: recognize perimeter.

Priority M.3.SR.MD.8

Efficiently, flexibly, and accurately solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

GRADE 4

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.4.DA.DS.1 Generate questions of interest to the students that elicit data, generate ideas based on the questions, and refine the question as necessary.

Collect and consider data.

M.4.DA.DS.2 Determine strategies for collecting and considering data in a variety of ways including with the use of technology, evaluate whether additional data that should be collected to completely address the investigative question.

Analyze the data.

Priority M.4.DA.DS.3

Critically analyze data visualizations, including tables, bar graphs, line plots, or spreadsheets to support a claim related to the investigative question. Ask whether the data collected sufficiently addresses the investigative question.

Interpret results.

Priority M.4.DA.DS.4

Interpret and communicate results, describing difference between groups, with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question.

Represent and interpret data.

M.4.DA.MD.4 Make a line plot to display a data set of measurements in fractions of a unit. Efficiently, flexibly, and accurately solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Quantity

Use the four operations with whole numbers to solve problems.

Priority M.4.Q.OA.2

Multiply or divide to efficiently, flexibly, and accurately solve real world word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Priority M.4.Q.OA.3

Efficiently, flexibly, and accurately solve multistep real world word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using visual models and equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using estimation strategies.

Gain familiarity with factors and multiples.

M.4.Q.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

M.4.Q.OA.5 Generate a number or shape pattern that follows a given rule. Identify and explain apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain why the numbers will continue to alternate in this way.*

Generalize place value understanding for multi-digit whole numbers.

Priority M.4.Q.NBT.2

Read, write, and compare multi-digit whole numbers using base-ten numerals, number names, and expanded form using the meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

M.4.Q.NBT.3 Use place value understanding of multi-digit whole numbers to generate estimates to any place less than or equal to 1,000,000 using a variety of estimation strategies.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Priority M.4.Q.NBT.4

Efficiently, flexibly, and accurately add and subtract multi-digit whole numbers using strategies or algorithms.

Priority M.4.Q.NBT.5

Efficiently, flexibly, and accurately multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Priority M.4.Q.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using multiple strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Relationships

Use the four operations with whole numbers to solve problems.

M.4.R.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent these comparison statements as multiplication equations.

Generalize place value understanding for multi-digit whole numbers.

M.4.R.NBT.1 Understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

Extend understanding of fraction equivalence and ordering.

M.4.R.NF.1 Demonstrate understanding of why a fraction is equivalent to another fraction by using visual fraction models (e.g., tape diagrams and number lines), with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Understand and use general principles to recognize and generate equivalent fractions.

Priority M.4.R.NF.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing them to a benchmark fraction. Understand that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Priority M.4.R.NF.3

Efficiently, flexibly, and accurately compose and decompose fractions with a numerator greater than 1 into unit fractions, including fractions greater than one or mixed numbers, to solve situations in context with addition and subtraction of fractions with like denominators.

- **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
- **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

M.4.R.NF.4 Flexibly apply and extend previous understandings of multiplication to multiply a fraction by a whole number using visual models in the context of word problems.

- **a.** Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).*
- **b.** Understand a multiple of *a/b* as a multiple of 1/*b,* and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5.*
- **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Understand decimal notation for fractions and compare decimal fractions.

Priority M.4.R.NF.5

Explore and explain using models, words, and/or numbers that a fraction with a denominator of 10 is an equivalent fraction with denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

Priority M.4.R.NF.6

Explore and explain decimal notation for fractions with denominators of 10 and 100 using models, words, and/or numbers. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

M.4.R.NF.7 Compare two decimals to hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, < and justify the conclusions by using multiple strategies and/or visual models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Priority M.4.R.MD.2

Use the four operations to efficiently, flexibly, and accurately solve real world word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using multiple visual models.

Geometric measurement: understand concepts of angles and measure angles.

M.4.R.MD.5 Demonstrate understanding of angles as geometric figures that are formed wherever two rays share a common endpoint and accurately measure an angle.

- **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
- **b.** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

M.4.R.MD.7 Demonstrate understanding that when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Efficiently, flexibly, and accurately solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Spatial Reasoning

Draw and identify lines and angles and classify shapes by properties of their lines and angles.

M.4.SR.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these features in two-dimensional figures.

M.4.SR.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.

M.4.SR.G.3 Recognize a line of symmetry for a two-dimensional figure as a line separating the figure such that the figure can be folded along the line into identical parts. Identify line-symmetric figures and draw lines of symmetry.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

M.4.SR.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec., and express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

M.4.SR.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Geometric measurement: understand concepts of angles and measure angles.

M.4.SR.MD.6 Measure angles in whole-number degrees using a protractor. Represent angles of specified measure.

GRADE 5

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.5.DA.DS.1 Generate questions of interest to the students that elicit data, generate ideas based on the questions, and refine the question as necessary. Pose statistical questions that can compare two variables withing a group, setting, or situation.

Collect and consider data.

M.5.DA.DS.2 Determine strategies for collecting and considering data in a variety of ways including with the use of technology. Understand that data may contain errors (missing values, etc.) and decisions have to be made on how to account for or resolve these issues.

Analyze the data.

Priority M.5.DA.DS.3

Critically analyze data visualizations, including tables, bar graphs, line plots, or spreadsheets to support a claim related to the investigative question. Compare and contrast different data visualizations to determine which transparently communicate results and interpretations.

Interpret results.

Priority M.5.DA.DS.4

Interpret and communicate results, describing difference between groups, with teacher guidance. Make a statement(s) about the data collected to support the answer to the investigative question. Describe the difference between two groups with different conditions.

Represent data.

M.5.DA.MD.2 Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Analyze patterns and relationships.

M.5.DA.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Quantity

Use equivalent fractions as a strategy to add and subtract fractions.

Priority M.5.Q.NF.1

Add and subtract fractions with unlike denominators (including mixed numbers) using flexible and efficient strategies, including replacing given fractions with equivalent fractions with like denominators. Justify using visual models (e.g., tape diagrams or number lines).

Priority M.5.Q.NF.2

Efficiently, flexibly, and accurately solve real world word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Priority M.5.Q.NF.3

Interpret a fraction as division, where a quantity (the numerator) is divided into equal parts (the denominator). Flexibly and efficiently solve real world word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. Assess the reasonableness of answers using estimation strategies. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Priority M.5.Q.NF.4

Apply and extend previous understandings of multiplication to efficiently, flexibly, and accurately multiply a fraction or whole number by a fraction.

- **a.** Interpret the product *(a/b) x q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b*. *For example, use a visual fraction model*

to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15.

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Priority M.5.Q.NF.5

Interpret multiplication as scaling (resizing) by estimating whether a product will be larger or smaller than a given factor based on the size of the other factor, without performing the indicated multiplication.

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b = (n x a)/(n x b)* to the effect of multiplying *a/b* by 1.

M.5.Q.NF.6 Efficiently and flexibly solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using fraction models or equations to represent the problem. Assess the reasonableness of answers using mental and estimation strategies.

Priority M.5.Q.NF.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions using fraction models and equations to represent the problem.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3)÷ 4 = 1/12 because (1/12) x 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Understand the place value system.

Priority M.5.Q.NBT.3

Read, write, and compare decimals to thousandths.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

M.5.Q.NBT.5 Efficiently, flexibly, and accurately multiply mult-digit whole numbers using strategies or algorithms.

Relationships

Write and interpret numerical expressions.

M.5.R.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

M.5.R.OA.2 Write expressions that record calculations with numbers and interpret numerical expressions without evaluating them . *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Understand the place value system.

M.5.R.NBT.1 Understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and of what it represents in the place to its left.

M.5.R.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.

M.5.R.NBT.4 Use place value understanding of decimals to generate estimates to any place using a variety of estimation strategies.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Priority M.5.R.NBT.6

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using strategies based on place value, properties of operations, and connected to the relationship between multiplication and division including rectangular arrays, partial quotients, and/or area models.

Priority M.5.R.NBT.7

Efficiently, flexibly, and accurately add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Spatial Reasoning

Graph points on the coordinate plane to solve real-world and mathematical problems.

M.5.SR.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system , with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Priority M.5.SR.G.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

M.5.SR.G.3 Demonstrate understanding that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

M.5.SR.G.4 Classify two-dimensional figures into categories based on properties.

Convert like measurement units within a given measurement system.

M.5.SR.MD.1 Convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real world problems. Assess the reasonableness of answers using mental and estimation strategies.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

M.5.SR.MD.3 Identify volume as an attribute of solid figures and understand concepts of volume measurement.

- **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

M.5.SR.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Priority M.5.SR.MD.5

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

GRADE 6

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.6.DA.DS.1 Formulate and recognize statistical investigative questions that are of interest to students to collect data from online sources and websites, smartphones, sensors, publicly available government agencies, and other modern devices.

Collect and consider data.

Priority M.6.DA.DS.2

Collect and record data with technology to identify and describe the characteristics of data sets. Understand that data can be collected (primary data) or existing data can be obtained from other sources (secondary data).

Analyze the data.

Priority M.6.DA.DS.3

Analyze data visualizations and describe measures of center and variability of quantitative data using appropriate displays (dot plots, boxplots). Describe key features of distributions for the variables including center, variability, and shape.

Interpret results.

M.6.DA.DS.4 Use statistical evidence from analyses to answer the statistical investigative question and communicate results with comprehensive answers with some teacher guidance.

Develop understanding of statistical variability.

M.6.DA.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

M.6.DA.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

M.6.DA.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

M.6.DA.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Priority M.6.DA.SP.5

Summarize numerical data sets in relation to their context.

- **a.** Reporting data points as the set of a number of observations.
- **b.** Describe what's being measured including how it was measured and its units of measurement.
- **c.** Calculate measures of center (mean and/or median) and variability (interquartile range and/or mean absolute deviation) of the data. Understand the shape of the data and identify any striking deviations (outliers) and connect these features to the context where the data came from.
- **d.** Communicate choice for selecting a given measure of center and variability and the connection to the shape of the data distribution related to the context in which the data were gathered.

Quantity

Compute efficiently, flexibly, and accurately with multi-digit numbers and find common factors and multiples.

M.6.Q.NS.2 Efficiently, flexibly, and accurately divide multi-digit numbers using strategies or algorithms.

M.6.Q.NS.3 Efficiently, flexibly, and accurately add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms for each operation.

Apply and extend previous understandings of numbers to the system of rational numbers.

M.6.Q.NS.5 Explain how positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Priority M.6.Q.NS.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to place any number (integer or rational, positive or negative) on the line (horizontal or vertical), and in the plane.

- **a.** Understand opposite signs of numbers as indicating locations on opposite sides of 0 on the number line, and recognize that, in situations like -(-3), the opposite of the opposite of a number is the number itself (in this example, 3).
- **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

M.6.Q.NS.7 Understand ordering and absolute value of positive and negative rational numbers.

- **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- **b.** Write, interpret, and explain order for rational numbers on a number line in real-world contexts, including the use of inequalities.
- **c.** Interpret absolute value as the distance a number is from zero on a number line, and understand the magnitude of absolute value in real world contexts like comparing temperatures, or understanding the size of a debt. *For example, for an account balance of -30 dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.*
- **d.** Distinguish comparisons of absolute value from statements about order.

Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

Priority M.6.R.RP.1

Explain the concept of a ratio and efficiently, flexibly, and accurately use ratio language to describe a ratio relationship between two quantities.

M.6.R.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

Priority M.6.R.RP.3

Efficiently, flexibly, and accurately demonstrate understanding of ratio and rate reasoning by solving real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations to find different ways to express the same ratio.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems (like price per item and constant speed)
c. Find percent of a quantity as a rate per 100 (a special ratio out of 100), including finding the whole, a part, and the percent as appropriate for a given context.
d. Use ratios to convert between different measurement units, like inches to feet, considering when it is appropriate to multiply or divide quantities.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Priority M.6.R.NS.1

Interpret and efficiently, flexibly, and accurately determine quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Compute efficiently, flexibly, and accurately with multi-digit numbers and find common factors and multiples.

M.6.R.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understandings of arithmetic to algebraic expressions.

M.6.R.EE.1 Efficiently, flexibly, and accurately write and evaluate numerical expressions involving whole-number exponents.

M.6.R.EE.2 Read, and evaluate expressions efficiently, flexibly, and accurately.
a. Write expressions in which letters stand for numbers to write general instructions like "subtract y from 5" as a mathematical expression (5 - *y*).
b. Break down expressions into smaller parts using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
c. Replace values for the variables (evaluating the expression) and use the Order of Operations where appropriate to solve problems including using real-world formulas, and formulas with whole-number exponents.

Priority M.6.R.EE.3

Apply the properties of operations (including the distributive property) efficiently, flexibly, and accurately to generate equivalent expressions.

M.6.R.EE.4 Identify whether two expressions are equivalent (including expressions with one or more variables) as both expressions will always yield the same outcome for any value of a given variable.

Reason about and solve one-variable equations and inequalities.

M.6.R.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Priority M.6.R.EE.6

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

M.6.R.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

M.6.R.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

Priority M.6.R.EE.9

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation.

Spatial Reasoning

Solve real-world and mathematical problems involving area, surface area, and volume.

M.6.SR.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by efficiently, flexibly, and accurately composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Priority M.6.SR.G.2

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to

find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

M.6.SR.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

M.6.SR.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Apply and extend previous understandings of numbers to the system of rational numbers.

Priority M.6.SR.NS.8

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate

GRADE 7

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

Priority M.7.DA.DS.1

Pose statistical investigative questions about a broader population using samples taken from the population.

Collect and consider data.

M.7.DA.DS.2 Understand information from a sample is valid only if the sample is representative of that population. Understand data can be used to make comparisons between different groups at one point in time and the same group over time.

Analyze the data.

M.7.DA.DS.3 Identify, determine, and interpret measures of center (mean and median) and measures of variability (range, interquartile range) to answer a statistically investigative question, summarizing the distribution of data using the measures of center and variability. Use reasoning about distributions to compare two groups based on the variables.

Interpret results.

Priority M.7.DA.DS.4

Acknowledge that looking beyond the data is feasible and recognize the uncertainty caused by sample-to-sample variability when making comparisons and/or conclusions from data to answer the investigative question.

Use random sampling to draw inferences about a population.

M.7.DA.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Priority M.7.DA.SP.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

M.7.DA.SP.3 Informally assess the degree of overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

M.7.DA.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Investigate chance processes and develop, use, and evaluate probability models.

M.7.DA.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Priority M.7.DA.SP.6

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability.

M.7.DA.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- ***a.*** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
- **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

M.7.DA.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- **a.** Understand the probability of a compound event is a fraction of the outcomes of the sample space.
- **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- **c.** Design and use a simulation to generate frequencies for compound events.

Quantity

Apply and extend previous understandings of operations with fractions.

M.7.Q.NS.1 Efficiently, flexibly, and accurately apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- **a.** Describe situations in which opposite quantities combine to make 0.
- **b.** Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- **d.** Apply properties of operations as strategies to add and subtract rational numbers.

Priority M.7.Q.NS.2

Efficiently, flexibly, and accurately apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- **b.** Understand integers can be divided as long as the divisor isn't zero, resulting in rational numbers.
- **c.** Apply properties of operations as strategies to multiply and divide rational numbers.
- **d.** Convert rational numbers into decimals using flexible, efficient, and accurate strategies, recognizing that the decimal form either ends in 0s or repeats eventually.

Draw, construct, and describe geometrical figures and describe the relationships between them.

M.7.Q.G.1 Efficiently, flexibly, and accurately solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing, and reproducing a scale drawing at a different scale.

Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

M.7.R.RP.1 Efficiently, flexibly, and accurately compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Priority M.7.R.RP.2

Recognize and represent proportional relationships between quantities.

- **a.** Decide whether the relationship between two quantities is proportional using equivalent ratios in a table, graphing on the coordinate plane to see of the graph is a straight line through origin.
- **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **c.** Write equations for proportional relationships.
- **d.** Analyze graphs to understand what the data points indicate about the real-world situation, focusing on points like (0, 0) and (1, *r*) where *r* is the unit rate.

M.7.R.RP.3 Efficiently, flexibly, and accurately use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Use properties of operations to generate equivalent expressions.

M.7.R.EE.1 Efficiently, flexibly, and accurately use properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Priority M.7.R.EE.2

Understand that rewriting an expression in different forms in a problem context can bring awareness to parts of the problem and how the quantities in it are related.

Efficiently, flexibly, and accurately solve real-life and mathematical problems using numerical and algebraic expressions and equations.

M.7.R.EE.3 Efficiently, flexibly, and accurately solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Priority M.7.R.EE.4

Use variables to represent quantities in a real-world or mathematical problem and write equations and inequalities to efficiently, flexibly, and accurately to solve problems by reasoning about the quantities.

- **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Compare solving the same problem algebraically vs. with arithmetic, explaining the steps involved in each approach.
- **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solutions of these inequalities and interpret them in context of the problem.

Spatial Reasoning

Draw, construct, and describe geometrical figures and describe the relationships between them.

M.7.SR.G.2 Draw geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

M.7.SR.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-world and mathematical problems involving area, surface area, and volume.

M.7.SR.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; informally show the relationship between the circumference and area of a circle.

M.7.SR.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Priority M.7.SR.G.6

Efficiently, flexibly, and accurately solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

GRADE 8

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.8.DA.DS.1 Formulate statistical investigative questions to articulate research topics and uncover patterns of association seen in bivariate categorical data, that multiple investigative questions may exist for a research topic and must take into account context.

Collect and consider data.

M.8.DA.DS.2 Understand how to interrogate the data to determine how the data were collected, from whom they were collected, what types of variables are in the data, how the variables were measured, and possible outcomes for the variables.

Analyze the data.

Priority M.8.DA.DS.3

Create data visualizations about a data set. Organize and present the data in appropriate ways, including in tables and scatter plots, and incorporate other relevant information that helps to tell a story and support a claim about the data.

Interpret results.

Priority M.8.DA.DS.4

Generalize beyond the sample providing statistical evidence for the conclusion, being sure to address limitations of the sample, evidenced in the data. Consider the reasonableness of the results.

Subdomain or category

M.8.DA.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

M.8.DA.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Priority M.8.DA.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

M.8.DA.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Quantity

Know that there are numbers that are not rational and approximate them by rational numbers.

M.8.Q.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers efficiently, flexibly, and accurately show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

M.8.Q.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Work with radicals and integer exponents.

M.8.Q.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Priority M.8.Q.EE.2

Use square roots and cube roots where p is a positive rational number. Use square root symbols to represent solutions to equations of the form $x^2 = p$. Evaluate square roots of perfect squares. Use cube root symbols to represent solutions to equations of the form $x^3 = p$ and evaluate cube roots of perfect cubes. Know that $\sqrt{2}$ is irrational.

M.8.Q.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

M.8.Q.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of

appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Relationships

Understand the connections between proportional relationships, lines, and linear equations.

M.8.R.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

M.8.R.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

Priority M.8.R.EE.7

Efficiently, flexibly, and accurately solve linear equations in one variable.

- **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- **b.** Solve linear equations with rational number coefficients where solution paths may require using the distributive property and combining like terms.

Priority M.8.R.EE.8

Analyze and efficiently, flexibly, and accurately solve pairs of simultaneous linear equations.

- **a.** Understand the solution to a system of linear equations is the point of intersection, because the intersection is a solution to both equations.
- **b.** Solve systems of linear equations using a variety of strategies (algebraically, graphically, numerically in tables, verbally, etc.)
- **c.** Solve real-world and mathematical problems leading to two linear equations in two variables.

Apply and extend previous understandings of arithmetic to algebraic expressions.

M.8.R.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

M.8.R.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

M.8.R.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the

area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.

Priority M.8.R.F.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

M.8.R.F.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described informally.

Spatial Reasoning

Understand congruence and similarity using physical models, transparencies, or geometry software.

M.8.SR.G.1 Verify experimentally the properties of rotations, reflections, and translations.

- **a.** Line and segment lengths are preserved in rotations, reflections, and translations.
- **b.** Angles are taken to angles of the same measure.
- **c.** Parallel lines are taken to parallel lines.

M.8.SR.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

M.8.SR.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Priority M.8.SR.G.4

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

M.8.SR.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

M.8.G.B.6 Efficiently, flexibly and accurately explain a proof of the Pythagorean Theorem and its converse.

Priority M.8.SR.G.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

M.8.SR.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving area, surface area, and volume.

M.8.SR.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

INTEGRATED MATH 1

A student's credit 1 and 2 math selection should align with their High School and Beyond Plan and be aligned with course equivalency to Algebra 1 or Geometry, or Integrated Math 1 or 2. OSPI acknowledges credit 1 and 2 equivalencies may be designed to address any combination of standards in this document, or additional Common Core Mathematics Standards not stated here, with increasing complexity and depth in each successive year.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

Priority M.HS.DA.DS.1

Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

M.HS.DA.DS.2 Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

Priority M.HS.DA.DS.3

Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

M.HS.DA.DS.4 Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand the concept of a function and use function notation.

M.HS.DA.FIF.1 Understand that a function has a domain (input, independent elements) and range (output, dependent elements), and assigns to each domain element exactly one range element. If f is a function and x is a value of its domain, then the output of f corresponds to the input x. The graph of f is the graph of the equation $y = f(x)$.

M.HS.DA.FIF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Summarize, represent, and interpret data on a single count or measurement variable.

Priority M.HS.DA.SID.1

Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.HS.DA.SID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Priority M.HS.DA.SID.3

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.HS.DA.SID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.HS.DA.SID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- **b.** [Not addressed in this course.]
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

Priority M.HS.DA.SID.7

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

M.HS.DA.SID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Priority M.HS.DA.SID.9
Distinguish between correlation and causation.

Quantity

Reason quantitatively and use units to solve problems.

Priority M.HS.Q.NQ.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Priority M.HS.Q.NQ.2
Define appropriate quantities for the purpose of descriptive modeling.

Priority M.HS.Q.NQ.3
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context within linear, exponential, and quadratic functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph linear, exponential, and quadratic functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.

M.HS.Q.FIF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions could be linear, exponential, or quadratic.

Create equations that describe numbers or relationships.

M.HS.Q.ACED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context within linear, quadratic, and exponential equations.

M.HS.Q.ACED.4 Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations within linear, quadratic, and exponential equations.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

Understand the concept of a function and use function notation.

M.HS.Q.FIF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology, within linear, exponential, and quadratic functions.

Relationships

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For a function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. Functions could be linear, exponential, or quadratic.

M.HS.R.FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions could be linear, exponential, or quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Create equations that describe numbers or relationships.

Priority M.HS.R.ACED.1

Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

Priority M.HS.R.ACED.2

Efficiently, flexibly, and accurately create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions in terms of context, including:

- **a.** Linear functions grow with equal differences over equal intervals and with exponential functions grow with equal factors over equal intervals.
- **b.** Recognizing constant rates per unit interval relative to another.
- **c.** Recognize contexts of growth or decay by a constant percent rate per unit interval relative to another.

M.HS.R.FLE.2 Efficiently, flexibly, and accurately construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically.

Interpret expressions for functions in terms of the situation they model.

Priority M.HS.R.FLE.5

Interpret the parameters in a linear or exponential function in terms of a context.

Understand solving equations as a process of reasoning and explain the reasoning.

Priority M.HS.R.AREI.1

Given that an original equation has a solution, efficiently, flexibly, and accurately select and use strategies to solve the equation, explaining each step, and construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

MHS.R.AREI.3 Efficiently, flexibly, and accurately solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

M.HS.R.AREI.5 Efficiently, flexibly, and accurately demonstrate that systems of two equations in two variables maintain the same solution set when one equation is replaced by the sum of that equation and a multiple of the other equation.

M.HS.R.AREI.6 Efficiently, flexibly, and accurately solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

M.HS.SR.AREI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, exponential, and quadratic.

M.HS.SR.AREI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.5 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.HS.SR.GGPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.HS.SR.GGPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Experiment with transformations in the plane.

Priority M.HS.SR.GCO.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.HS.SR.GCO.2 Efficiently, flexibly, and accurately represent transformations in the plane, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.HS.SR.GCO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the

rotations and reflections that carry it onto itself.

Priority M.HS.SR.GCO.4

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

M.HS.SR.GCO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Efficiently, flexibly and accurately specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

Priority M.HS.SR.GCO.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.HS.SR.GCO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.HS.SR.GCO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Make geometric constructions.

M.HS.SR.GCO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

M.HS.SR.GCO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

INTEGRATED MATH 2

A student's credit 1 and 2 math selection should align with their High School and Beyond Plan and be aligned with course equivalency to Algebra 1 or Geometry, or Integrated Math 1 or 2. OSPI acknowledges credit 1 and 2 equivalencies may be designed to address any combination of standards in this document, or additional Common Core Mathematics Standards not stated here, with increasing complexity and depth in each successive year.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data

supports, taking into account correlation versus causation.

Understand independence and conditional probability and use them to interpret data.

Priority M.HS.DA.SCP.1

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

M.HS.DA.SCP.2 Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

M.HS.DA.SCP.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B),}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

M.HS.DA.SCP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Priority M.HS.DA.SCP.5

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events.

M.HS.DA.SCP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

M.HS.DA.SCP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Quantity

Understand similarity in terms of similarity transformations.

M.HS.Q.GSRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Define trigonometric ratios and solve problems involving right triangles.

M.HS.Q.GSRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.HS.Q.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

M.HS.Q.GSRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context within linear, exponential, and quadratic functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite it such as the difference of squares, compound interest, and others.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph linear, exponential, and quadratic functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, and integer constants for exponential growth and decay, rewrite a function in different equivalent forms.

- **a.** Factor quadratic functions to verify and explain various properties on graphs, such as zeros and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions.

Priority M.HS.Q.FIF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions could be linear, exponential, or quadratic.

Priority M.HS.Q.ACED.4

Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations within linear, quadratic, and exponential equations.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k, k\,f(x), f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology, within linear, exponential, and quadratic functions.

Extend the properties of exponents to rational exponents.

M.HS.Q.NRN.1 Efficiently, flexibly, and accurately explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values using a variety of strategies, allowing for a notation for radicals in terms of rational exponents.

M.HS.Q.NRN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

M.HS.Q.NRN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials, limited to quadratic equations.

Relationships

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For a function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. Functions could be linear, exponential, or quadratic.

Priority M.HS.R.FIF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions could be linear, exponential, or quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Write expressions in equivalent forms to solve problems.

Priority M.HS.R.ASSE.3

Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions (linear, quadratic, exponential)

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** [Not addressed in this course.]
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Create equations that describe numbers or relationships.

Priority M.HS.R.ACED.1

Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** [Not addressed in this course.]
- **b.** Solve by inspection (e.g., for $x^2 = 49$), taking square roots, and factoring, as appropriate to the initial equation.

Solve systems of equations.

M.HS.R.AREI.7 Efficiently, flexibly, and accurately solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Prove geometric theorems.

M.HS.R.GCO.9 Efficiently, flexibly, and accurately prove theorems about lines and angles: vertical angles, transversals, alternate interior and exterior angles, perpendicular bisectors, etc.

M.HS.R.GCO.10 Efficiently, flexibly, and accurately prove theorems about triangles: interior angles, base angles, segments joining midpoint of two sides, and medians of a triangle.

M.HS.R.GCO.11 Efficiently, flexibly, and accurately prove theorems about parallelograms: congruence of opposite sides and opposite angles, properties of diagonals.

Prove theorems involving similarity.

M.HS.R.GSRT.4 Efficiently, flexibly, and accurately prove theorems about triangles including proportionality, triangle similarity, and the Pythagorean Theorem.

Priority M.HS.R.GSRT.5

Efficiently, flexibly, and accurately use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Understand and apply theorems about circles.

M.HS.R.GC.1 Efficiently, flexibly, and accurately prove that all circles are similar.

M.HS.R.GC.2 Identify and describe relationships among inscribed angles, radii, and chords, including how angles formed inside the circle, the circle's radius, and line segments within the circle are related. Understand special cases including angles formed by diameters and how the circle's edge interacts with its radius.

Find arc lengths and areas of sectors of circles.

M.HS.R.GC.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Spatial Reasoning

Understand and apply theorems about circles.

M.HS.SR.GC.3 Construct the inscribed and circumscribed circles of a triangle and efficiently, flexibly, and accurately prove properties of angles for a quadrilateral inscribed in a circle.

Understand similarity in terms of similarity transformations.

Priority M.HS.SR.GSRT.1

Verify experimentally how lines are affected by the center of dilation and how the scale factor changes line segments.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Priority M.HS.SR.GSRT.2

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs

of sides.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem.

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

Explain volume formulas and use them to solve problems.

M.HS.SR.GGMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Priority M.HS.SR.GGMD.3

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations.

Priority M.HS.SR.GMG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Priority M.HS.SR.GMG.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Priority M.HS.SR.GMG.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

INTEGRATED MATH 3

A student's third credit of math should be based on the student's interest and their High School and Beyond Plan. Third credit math courses address math standards with increased complexity and depth from previous math courses. OSPI acknowledges third credit math courses may be designed to address any combination of the standards in this document.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand and evaluate random processes underlying statistical experiments.

M.HS.DA.SIC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

M.HS.DA.SIC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Priority M.HS.DA.SIC.3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Priority M.HS.DA.SIC.4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

M.HS.DA.SIC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Priority M.HS.DA.SIC.6

Evaluate reports based on data.

Summarize, represent, and interpret data on a single count or measurement variable.

M.HS.DA.SID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Priority M.HS.DA.SID.6

Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Models may include any family of functions.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Quantity

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context in any family of functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure of an expression to identify ways to rewrite it.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

M.HS.Q.FBF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

M.HS.Q.ASSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.

Analyze functions using different representations.

Priority M.HS.Q.FIF.7

Graph families of functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.
- **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **d.** [Not addressed in this course.]
- **e.** Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Factor quadratic functions and completing the square to show zeros, extreme values and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions, including non-integer constants for time when applicable in contexts of exponential growth or decay.

Priority M.HS.Q.FIF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology.

Create equations that describe numbers or relationships.

Priority M.HS.Q.ACED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. May include any families of functions.

Priority M.HS.Q.ACED.4

Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. May include any families of functions.

Perform arithmetic operations with complex numbers.

M.HS.Q.NCN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

M.HS.Q.NCN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

M.HS.Q.NCN.7 Solve quadratic equations with real coefficients that have complex solutions.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

M.HS.Q.AAPR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

M.HS.Q.AAPR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

M.HS.Q.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.

M.HS.Q.AAPR.6 Efficiently, flexibly, and accurately rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x)+r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Relationships

Create equations that describe numbers or relationships.

M.HS.R.ACED.1 Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Write expressions in equivalent forms to solve problems.

M.HS.R.ASSE.3 Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4
For any function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. May include any families of functions, and relative maximums and minimums, end behavior, and periodicity.

Priority M.HS.R.FIF.5
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a non-linear function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Build new functions from existing functions.

M.HS.R.FBF.4 Find inverse functions through focus on relationships between inputs and outputs.

- **a.** Solve and equation of the form $f(x) = c$ that has an inverse and write an equation for the inverse.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

Priority M.HS.SR.AREI.11
Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, polynomial, rational, absolute value, exponential, and logarithmic functions.

Extend the domain of trigonometric functions using the unit circle.

M.HS.SR.FTF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

M.HS.SR.FTF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

Priority M.HS.SR.FTF.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Prove and apply trigonometric identities.

M.HS.SR.FTF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$, given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

M.HS.SR.GGPE.2 Derive the equation of a parabola given a focus and directrix.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

GEOMETRY

A student's credit 1 and 2 math selection should align with their High School and Beyond Plan and be aligned with course equivalency to Algebra 1 or Geometry, or Integrated Math 1 or 2. OSPI acknowledges credit 1 and 2 equivalencies may be designed to address any combination of standards in this document, or additional Common Core Mathematics Standards not stated here, with increasing complexity and depth in each successive year.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data

supports, taking into account correlation versus causation.

Understand independence and conditional probability and use them to interpret data.

Priority M.HS.DA.SCP.1

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

M.HS.DA.SCP.2 Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

M.HS.DA.SCP.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B),}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

M.HS.DA.SCP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Priority M.HS.DA.SCP.5

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events.

M.HS.DA.SCP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

M.HS.DA.SCP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Quantity

Understand similarity in terms of similarity transformations.

M.HS.Q.GSRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Define trigonometric ratios and solve problems involving right triangles.

M.HS.Q.GSRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.HS.Q.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

M.HS.Q.GSRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Relationships

Solve real-world and mathematical problems involving area, surface area, and volume.

M.HS.R.GCO.9 Efficiently, flexibly, and accurately prove theorems about lines and angles: vertical angles, transversals, alternate interior and exterior angles, perpendicular bisectors, etc.

M.HS.R.GCO.10 Efficiently, flexibly, and accurately prove theorems about triangles: interior angles, base angles, segments joining midpoint of two sides, and medians of a triangle.

M.HS.R.GCO.11 Efficiently, flexibly, and accurately prove theorems about parallelograms: congruence of opposite sides and opposite angles, properties of diagonals.

Prove theorems involving similarity.

M.HS.R.GSRT.4 Efficiently, flexibly, and accurately prove theorems about triangles including proportionality, triangle similarity, and the Pythagorean Theorem.

Priority M.HS.R.GSRT.5

Efficiently, flexibly, and accurately use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Understand and apply theorems about circles.

M.HS.R.GC.1 Efficiently, flexibly, and accurately prove that all circles are similar.

M.HS.R.GC.2 Identify and describe relationships among inscribed angles, radii, and chords, including how angles formed inside the circle, the circle's radius, and line segments within the circle are related. Understand special cases including angles formed by diameters and how the circle's edge interacts with its radius.

Find arc lengths and areas of sectors of circles.

M.HS.R.GC.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Spatial Reasoning

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.5 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.HS.SR.GGPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.HS.SR.GGPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Experiment with transformations in the plane.

Priority M.HS.SR.GCO.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.HS.SR.GCO.2 Efficiently, flexibly, and accurately represent transformations in the plane, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.HS.SR.GCO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Priority M.HS.SR.GCO.4

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

M.HS.SR.GCO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Efficiently, flexibly, and accurately specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

Priority M.HS.SR.GCO.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.HS.SR.GCO.7 Use the definition of congruence in terms of rigid motions to show that two

triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.HS.SR.GCO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Make geometric constructions.

M.HS.SR.GCO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

M.HS.SR.GCO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Understand and apply theorems about circles.

M.HS.SR.GC.3 Construct the inscribed and circumscribed circles of a triangle, and efficiently, flexibly, and accurately prove properties of angles for a quadrilateral inscribed in a circle.

Understand similarity in terms of similarity transformations.

Priority M.HS.SR.GSRT.1

Verify experimentally how lines are affected by the center of dilation and how the scale factor changes line segments.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Priority M.HS.SR.GSRT.2

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Determine the equation of a circle of given center and radius using the Pythagorean Theorem.

Explain volume formulas and use them to solve problems.

M.HS.SR.GGMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Priority M.HS.SR.GGMD.3

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations.

Priority M.HS.SR.GMG.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Priority M.HS.SR.GMG.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Priority M.HS.SR.GMG.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

ALGEBRA 1

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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

Priority M.HS.DA.DS.1

Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

M.HS.DA.DS.2 Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

Priority M.HS.DA.DS.3

Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

M.HS.DA.DS.4 Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation

Understand the concept of a function and use function notation.

M.HS.DA.FIF.1 Understand that a function has a domain (input, independent elements) and range (output, dependent elements), and assigns to each domain element exactly one range element. If f is a function and x is a value of its domain, then the output of f corresponds to the input x. The graph of f is the graph of the equation $y = f(x)$.

M.HS.DA.FIF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

Summarize, represent, and interpret data on a single count or measurement variable.

Priority M.HS.DA.SID.1

Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.HS.DA.SID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Priority M.HS.DA.SID.3

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.HS.DA.SID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.HS.DA.SID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- **b.** [Not addressed in this course.]
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

Priority M.HS.DA.SID.7

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

M.HS.DA.SID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

Priority M.HS.DA.SID.9
Distinguish between correlation and causation.

Quantity

Reason quantitatively and use units to solve problems.

Priority M.HS.Q.NQ.1
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Priority M.HS.Q.NQ.2
Define appropriate quantities for the purpose of descriptive modeling.

Priority M.HS.Q.NQ.3
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context within linear, exponential, and quadratic functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite it including the difference of squares, compound interest and others.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph linear, exponential, and quadratic functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Factor quadratic functions to verify and explain various properties on graphs, such as zeros and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions.

M.HS.Q.FIF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions could be linear, exponential, or quadratic.

Create equations that describe numbers or relationships.

M.HS.Q.ACED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context within linear, quadratic, and exponential equations.

M.HS.Q.ACED.4 Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations within linear, quadratic, and exponential equations.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

Understand the concept of a function and use function notation.

M.HS.Q.FIF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology, within linear, exponential, and quadratic functions.

Extend the properties of exponents to rational exponents.

M.HS.Q.NRN.1 Efficiently, flexibly, and accurately explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values using a variety of strategies, allowing for a notation for radicals in terms of rational exponents.

M.HS.Q.NRN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

M.HS.Q.NRN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials., limited to quadratic equations.

Relationships

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For a function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. Functions could be linear, exponential, or quadratic.

M.HS.R.FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions could be linear, exponential, or quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Write expressions in equivalent forms to solve problems.

Priority M.HS.R.ASSE.3

Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions (linear, quadratic, exponential)

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** [Not addressed in this course.]
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Create equations that describe numbers or relationships.

Priority M.HS.R.ACED.1

Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

Priority M.HS.R.ACED.2

Efficiently, flexibly, and accurately create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions in terms of context, including:

- **a.** Linear functions grow with equal differences over equal intervals and with exponential functions grow with equal factors over equal intervals.
- **b.** Recognizing constant rates per unit interval relative to another.
- **c.** Recognize contexts of growth or decay by a constant percent rate per unit interval relative to another.

M.HS.R.FLE.2 Efficiently, flexibly, and accurately construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly and quadratically.

Interpret expressions for functions in terms of the situation they model.

Priority M.HS.R.FLE.5

Interpret the parameters in a linear or exponential function in terms of a context.

Understand solving equations as a process of reasoning and explain the reasoning.

Priority M.HS.R.AREI.1

Given that an original equation has a solution, efficiently, flexibly, and accurately select and use strategies to solve the equation, explaining each step, and construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

M.HS.R.AREI.3 Efficiently, flexibly, and accurately solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** [Not addressed in this course.]
- **b.** Solve by inspection (e.g., for $x^2 = 49$), taking square roots, and factoring, as appropriate to the initial equation.

Solve systems of equations.

M.HS.R.AREI.5 Efficiently, flexibly, and accurately demonstrate that systems of two equations in two variables maintain the same solution set when one equation is replaced by the sum of that equation and a multiple of the other equation.

M.HS.R.AREI.6 Efficiently, flexibly, and accurately solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

M.HS.R.AREI.7 Efficiently, flexibly, and accurately solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

M.HS.SR.AREI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, exponential, and quadratic.

M.HS.SR.AREI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

ALGEBRA 2

A student's third credit of math should be based on the student's interest and their High School and Beyond Plan. Third credit math courses address math standards with increased complexity and depth from previous math courses. OSPI acknowledges third credit math courses may be designed to address any combination of the standards in this document.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

Priority M.HS.DA.DS.2

Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

Priority M.HS.DA.DS.4

Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation.

Understand and evaluate random processes underlying statistical experiments.

M.HS.DA.SIC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

M.HS.DA.SIC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Priority M.HS.DA.SIC.3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Priority M.HS.DA.SIC.4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

M.HS.DA.SIC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Priority M.HS.DA.SIC.6

Evaluate reports based on data.

Summarize, represent, and interpret data on a single count or measurement variable.

M.HS.DA.SID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Priority M.HS.DA.SID.6

Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Models may include any family of functions.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Quantity

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context in any family of functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.

M.HS.Q.FBF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model linear and exponential situations, and translate between the two forms.

Write expressions in equivalent forms to solve problems.

M.HS.Q.ASSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.

Analyze functions using different representations.

Priority M.HS.Q.FIF.7

Graph families of functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.
- **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **d.** [Not addressed in this course.]
- **e.** Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Factor quadratic functions and completing the square to show zeros, extreme values and symmetry in terms of a context.

b. Use properties of exponents to interpret expressions for exponential functions, including non-integer constants for time when applicable in contexts of exponential growth or decay.

Priority M.HS.Q.FIF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology.

Create equations that describe numbers or relationships.

Priority M.HS.Q.ACED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. May include any families of functions.

Priority M.HS.Q.ACED.4

Efficiently, flexibly, and accurately rearrange formulas with multiple quantities of interests to highlight and contextualize a specific quantity, using the same reasoning as in solving equations. May include any families of functions.

Perform arithmetic operations with complex numbers.

M.HS.Q.NCN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

M.HS.Q.NCN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

M.HS.Q.NCN.7 Solve quadratic equations with real coefficients that have complex solutions.

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

M.HS.Q.AAPR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

M.HS.Q.AAPR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial

M.HS.Q.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.

M.HS.Q.AAPR.6 Efficiently, flexibly, and accurately rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x)+r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Relationships

Create equations that describe numbers or relationships.

M.HS.R.ACED.1 Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation, and use them to solve problems.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Write expressions in equivalent forms to solve problems.

M.HS.R.ASSE.3 Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Interpret functions that arise in applications in terms of the context.

Priority M.HS.R.FIF.4

For any function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. May include any families of functions, and relative maximums and minimums, end behavior, and periodicity.

Priority M.HS.R.FIF.5

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a non-linear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Build new functions from existing functions.

M.HS.R.FBF.4 Find inverse functions through focus on relationships between inputs and outputs.

- **a.** Solve and equation of the form *f(x) = c* that has an inverse and write an equation for the inverse.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Spatial Reasoning

Represent and solve equations and inequalities graphically.

Priority M.HS.SR.AREI.11

Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ find the solutions approximately, e.g., using technology to graph the functions, make tables of values,

or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, polynomial, rational, absolute value, exponential, and logarithmic functions.

Extend the domain of trigonometric functions using the unit circle.

M.HS.SR.FTF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle

M.HS.SR.FTF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Priority M.HS.SR.FTF.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

M.HS.SR.FTF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$, given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

Expressing geometric properties with equations.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem, and complete the square to determine the center and radius of a circle from the equation.

M.HS.SR.GGPE.2 Derive the equation of a parabola given a focus and directrix.

HIGH SCHOOL CREDIT 3

A student's third credit of math should be based on the student's interest and their High School and Beyond Plan. Third credit math courses address math standards with increased complexity and depth from previous math courses. OSPI acknowledges third credit math courses may be designed to address any combination of the standards in this document.

Note: Plus standards **(+)** are addressed in credits 3 and 4 mathematics in courses aligned with a student's High School and Beyond Plan.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with math.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Data Analysis

Formulate statistical investigative questions.

M.HS.DA.DS.1 Formulate multivariable statistical investigative questions and determine how data can be collected and provide an answer, consider causality and prediction when posing the question.

Collect and consider data.

M.HS.DA.DS.2 Understand the issues of bias and confounding variables when collecting data and their impact on interpretation. Understand practices for collecting and handling data, including sensitive information and concerns for privacy and how that may affect data collection.

Analyze the data.

M.HS.DA.DS.3 Create and analyze data sets using technology to clean, sort, or filter data. Summarize, and describe relationships between quantitative variables using data displays including but not limited to scatter plots, regressions, histograms, and boxplots.

Interpret results.

M.HS.DA.DS.4 Acknowledge the presence of missing data values and understand how missing values may add bias to analysis and interpretation. Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports, taking into account correlation versus causation

Understand the concept of a function and use function notation.

M.HS.DA.FIF.1 Understand that a function has a domain (input, independent elements) and range (output, dependent elements), and assigns to each domain element exactly one range element. If f is a function and x is a value of its domain, then the output of f corresponds to the input x. The graph of f is the graph of the equation $y = f(x)$.

M.HS.DA.FIF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Summarize, represent, and interpret data on a single count or measurement variable.

M.HS.DA.SID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.HS.DA.SID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

M.HS.DA.SID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

M.HS.DA.SID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.HS.DA.SID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

M.HS.DA.SID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related. Solve problems in context by fitting functions to the data and explaining trends and relationships within the data.

- **a.** When fitting a function to the data, use given functions or choose a function suggested by the context. Models may include any family of functions.
- **b.** Informally assess the fit of a function by plotting and analyzing residuals.
- **c.** Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

M.HS.DA.SID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

M.HS.DA.SID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

M.HS.DA.SID.9 Distinguish between correlation and causation.

Understand and evaluate random processes underlying statistical experiments.

M.HS.DA.SIC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

M.HS.DA.SIC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

M.HS.DA.SIC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

M.HS.DA.SIC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

M.HS.DA.SIC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant

M.HS.DA.SIC.6 Evaluate reports based on data

Understand independence and conditional probability and use them to interpret data.

M.HS.DA.SCP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

M.HS.DA.SCP.2 Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

M.HS.DA.SCP.3 Understand the conditional probability of A given B as $\frac{P(A \text{ and } B),}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

M.HS.DA.SCP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

M.HS.DA.SCP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events.

M.HS.DA.SCP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

M.HS.DA.SCP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

M.HS.DA.SCP.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

M.HS.DA.SCP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Calculate expected values and use them to solve problems.

M.HS.DA.SMD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

M.HS.DA.SMD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

M.HS.DA.SMD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

M.HS.DA.SMD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

Use probability to evaluate outcomes of decisions.

M.HS.DA.SMD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

- **a.** Find the expected payoff for a game of chance.
- **b.** Evaluate and compare strategies on the basis of expected values.

M.HS.DA.SMD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

M.HS.DA.SMD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Quantity

Reason quantitatively and use units to solve problems.

M.HS.Q.NQ.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

M.HS.Q.NQ.2 Define appropriate quantities for the purpose of descriptive modeling.

M.HS.Q.NQ.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions.

M.HS.Q.ASSE.1 Interpret expressions that represent a quantity in terms of its context in any family of functions.

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

M.HS.Q.ASSE.2 Use the structure or pattern of an expression to identify ways to rewrite.

Analyze functions using different representations.

M.HS.Q.FIF.7 Graph families of functions from their symbolic forms by hand or with technology, and identify key features including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries, and end behavior.

- **a.** Including intercepts, maxima, and minima.
- **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **d.** Graph logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **e.** **(+)** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

M.HS.Q.FIF.8 Efficiently, flexibly, and accurately, write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Factor quadratic functions and completing the square to show zeros, extreme values and symmetry in terms of a context.
- **b.** Use properties of exponents to interpret expressions for exponential functions, including non-integer constants for time when applicable in contexts of exponential growth or decay.

M.HS.Q.FIF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

Create equations that describe numbers or relationships.

M.HS.Q.ACED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. May include any families of functions.

M.HS.Q.ACED.4 Efficiently, flexibly, and accurately rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. May include any families of functions.

Build a function that models a relationship between two quantities.

M.HS.Q.FBF.1 Efficiently, flexibly, and accurately write a function that describes a relationship between two quantities, including linear and exponential relationships, and arithmetic and geometric sequences in context.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations.
- **c.** **(+)** Compose functions.

M.HS.Q.FBF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model linear and exponential situations, and translate between the two forms.

Understand the concept of a function and use function notation.

M.HS.Q.FIF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Write expressions in equivalent forms to solve problems.

M.HS.Q.ASSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.

Build new functions from existing functions.

M.HS.Q.FBF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Using a variety of strategies, experiment with cases and illustrate an explanation of the effects on the graph using technology.

Extend the properties of exponents to rational exponents.

M.HS.Q.NRN.1 Efficiently, flexibly, and accurately explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values using a variety of strategies, allowing for a notation for radicals in terms of rational exponents.

M.HS.Q.NRN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

M.HS.Q.NRN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations with complex numbers.

M.HS.Q.NCN.1 Know there is a complex number i such that i2 = -1, and every complex number has the form a + bi with a and b real.

M.HS.Q.NCN.2 Use the relation i2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

M.HS.Q.NCN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

M.HS.Q.NCN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.

M.HS.Q.NCN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

M.HS.Q.NCN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

M.HS.Q.NCN.7 Solve quadratic equations with real coefficients that have complex solutions.

M.HS.Q.NCN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

M.HS.Q.NCN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Represent and model with vector quantities.

M.HS.Q.NVM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., $v, |v|, ||v||, \vec{v}$).

M.HS.Q.NVM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

M.HS.Q.NVM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

M.HS.Q.NVM.4 (+) Add and subtract vectors.

- **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- **c.** Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.

M.HS.Q.NVM.5 (+) Multiply a vector by a scalar.

- **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.
- **b.** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Perform operations on matrices and use matrices in applications.

M.HS.Q.NVM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

M.HS.Q.NVM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

M.HS.Q.NVM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

M.HS.Q.NVM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties.

M.HS.Q.NVM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

M.HS.Q.NVM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

M.HS.Q.NVM.12 (+) Work with 2 x 2 matrices as

transformations of the plane and interpret the absolute value of the determinant in terms of area.

Solve systems of equations.

M.HS.Q.AREI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

M.HS.Q.AREI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Perform arithmetic operations on polynomials.

M.HS.Q.AAPR.1 Efficiently, flexibly, and accurately demonstrate that polynomials form a system with a structure similar to that of integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

M.HS.Q.AAPR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

M.HS.Q.AAPR.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

M.HS.Q.AAPR.4 Prove polynomial identities and use them to describe numerical relationships.

M.HS.Q.AAPR.5 (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

M.HS.Q.AAPR.6 Efficiently, flexibly, and accurately rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x) + r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

M.HS.Q.AAPR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Understand similarity in terms of similarity transformations.

M.HS.Q.GSRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Define trigonometric ratios and solve problems involving right triangles.

M.HS.Q.GSRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.HS.Q.GSRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

M.HS.Q.GSRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Relationships

Interpret functions that arise in applications in terms of the context.

M.HS.R.FIF.4 For any function that models a relationship between two quantities in context of the relationship, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and symmetries. May include any families of functions, and relative maximums and minimums, end behavior, and periodicity.

M.HS.R.FIF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes in context. Functions can include polynomial, radical, rational, logarithms, absolute value, piecewise, trigonometric, and increasingly complex linear, exponential, and quadratic relationships.

M.HS.R.FIF.6 Calculate and interpret the average rate of change of a non-linear function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Create equations that describe numbers or relationships.

M.HS.R.ACED.1 Efficiently, flexibly, and accurately create equations and inequalities in one variable that model a situation and use them to solve problems.

M.HS.R.ACED.2 Efficiently, flexibly, and accurately create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Write expressions in equivalent forms to solve problems.

M.HS.R.ASSE.3 Efficiently, flexibly, and accurately rewrite expressions to equivalent forms that are suitable for the purpose of revealing and explaining a specific property about those functions.

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions.

Solve equations and inequalities in one variable.

M.HS.R.AREI.4 Solve quadratic equations in one variable using a process of identifying solutions as appropriate to the initial form of the equation.

- **a.** Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions in terms of context, including:

- **a.** Linear functions grow with equal differences over equal intervals and with exponential functions grow with equal factors over equal intervals.
- **b.** Recognizing constant rates per unit interval relative to another.
- **c.** Recognize contexts of growth or decay by a constant percent rate per unit interval relative to another.

M.HS.R.FLE.2 Efficiently, flexibly, and accurately construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

M.HS.R.FLE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly and quadratically, or as a polynomial function.

Interpret expressions for functions in terms of the situation they model.

M.HS.R.FLE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.1 Given that an original equation has a solution, efficiently, flexibly, and accurately select and use strategies to solve the equation, explaining each step, and construct a viable argument to justify a solution method.

M.HS.R.AREI.3 Efficiently, flexibly, and accurately solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

M.HS.R.AREI.5 Efficiently, flexibly, and accurately demonstrate that systems of two equations in two variables maintain the same solution set when one equation is replaced by the sum of that

equation and a multiple of the other equation.

M.HS.R.AREI.6 Efficiently, flexibly, and accurately solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

M.HS.R.AREI.7 Efficiently, flexibly, and accurately solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Understand solving equations as a process of reasoning and explain the reasoning.

M.HS.R.AREI.2 Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Build new functions from existing functions.

M.HS.R.FBF.4 Find inverse functions through focus on relationships between inputs and outputs.

- **a.** Solve an equation of the form f(x) = c that has an inverse and write an equation for the inverse.
- **b.** **(+)** Verify by composition that one function is the inverse of another.
- **c.** **(+)** Read values of an inverse function from a graph or a table, given that the function has an inverse.
- **d.** **(+)** Produce an invertible function from a non-invertible function by restricting the domain.

M.HS.R.FBF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.HS.R.FLE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Prove geometric theorems.

M.HS.R.GCO.9 Efficiently, flexibly, and accurately prove theorems about lines and angles: vertical angles, transversals, alternate interior and exterior angles, perpendicular bisectors, etc.

M.HS.R.GCO.10 Efficiently, flexibly, and accurately prove theorems about triangles: interior angles, base angles, segments joining midpoint of two sides, and medians of a triangle.

M.HS.R.GCO.11 Efficiently, flexibly, and accurately prove theorems about parallelograms: congruence of opposite sides and opposite angles, properties of diagonals.

Prove theorems involving similarity.

M.HS.R.GSRT.4 Efficiently, flexibly, and accurately prove theorems about triangles including proportionality, triangle similarity, and the Pythagorean Theorem.

M.HS.R.GSRT.5 Efficiently, flexibly, and accurately use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Apply trigonometry to general triangles.

M.HS.R.GSRT.9 (+) Derive the formula $A = 1/2\ ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

M.HS.R.GSRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

M.HS.R.GSRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Understand and apply theorems about circles.

M.HS.R.GC.1 Efficiently, flexibly, and accurately prove that all circles are similar.

M.HS.R.GC.2 Identify and describe relationships among inscribed angles, radii, and chords, including how angles formed inside the circle, the circle's radius, and line segments within the circle are related. Understand special cases including angles formed by diameters and how the circle's edge interacts with its radius.

M.HS.R.GC.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Spatial Reasoning

Extend the domain of trigonometric functions using the unit circle.

M.HS.SR.FTF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

M.HS.SR.FTF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

M.HS.SR.FTF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

M.HS.SR.FTF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodicity phenomena with trigonometric functions.

M.HS.SR.FTF.5 Choose trigonometric functions to model periodic phenomena with specified

amplitude, frequency, and midline.

M.HS.SR.FTF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

M.HS.SR.FTF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

M.HS.SR.FTF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$, given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

M.HS.SR.FTF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Translate between the geometric description and the equation for a conic section.

M.HS.SR.GGPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem, and completing the square to determine the center of a circle from the equation.

M.HS.SR.GGPE.2 Derive the equation of a parabola given a focus and directrix.

M.HS.SR.GGPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.4 Use coordinates to prove simple geometric theorems algebraically.

M.HS.SR.GGPE.5 Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.HS.SR.GGPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.HS.SR.GGPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Experiment with transformations in the plane.

M.HS.SR.GCO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.HS.SR.GCO.2 Efficiently, flexibly, and accurately represent transformations in the plane, e.g.

transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

M.HS.SR.GCO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

M.HS.SR.GCO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

M.HS.SR.GCO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Efficiently, flexibly, and accurately specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

M.HS.SR.GCO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.HS.SR.GCO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

M.HS.SR.GCO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Make geometric constructions.

M.HS.SR.GCO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

M.HS.SR.GCO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Understand and apply theorems about circles.

M.HS.SR.GC.3 Construct the inscribed and circumscribed circles of a triangle and efficiently, flexibly, and accurately prove properties of angles for a quadrilateral inscribed in a circle.

M.HS.SR.GC.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Understand similarity in terms of similarity transformations.

M.HS.SR.GSRT.1 Verify experimentally how lines are affected by the center of dilation and how the

scale factor changes line segments.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

M.HS.SR.GSRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Explain volume formulas and use them to solve problems.

M.HS.SR.GGMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

M.HS.SR.GGMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

M.HS.SR.GGMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three-dimensional objects.

M.HS.SR.GGMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Represent and solve equations and inequalities graphically.

M.HS.SR.AREI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

M.HS.SR.AREI.11 Using a variety of strategies explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, polynomial, rational, absolute value, exponential, and logarithmic functions.

M.HS.SR.AREI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Apply geometric concepts in modeling situations.

M.HS.SR.GMG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

M.HS.SR.GMG.2 Apply concepts of density based on area and volume in modeling situations (e.g.,

persons per square mile, BTUs per cubic foot).

M.HS.SR.GMG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

APPENDIX A: Uplifting the Standards for Mathematical Practice (SMPs)

The Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They can analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that consider the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of

two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the math they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving mathematical problems. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels can identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They can use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers

with a degree of precision appropriate for the problem context. In elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to notice a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 - 3(x - y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and shortcuts. Upper elementary students might notice when dividing 25 by 11, they repeat the same calculations again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Using the Standards for Mathematical Practice

Throughout WA Math 2026, students are encouraged to utilize multiple ways of thinking and doing math, and to reflect on the reasonableness of their answers. Focusing on these practices increases students' understanding of the concepts offered in the early grades for greater success in later grades. An example of this can be found in a move from "the standard algorithm" toward "a strategy or algorithm" a move that centers the many ways to efficiently solve a problem mathematically. Shown in this 6th grade standard:

> Old **6.NS.B.3**: "Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation."
>
> New **M.6.Q.NS.3**: "Efficiently, flexibly, and accurately add, subtract, multiply, and divide multi-digit decimals using strategies or algorithms for each operation."

The change from "fluently" to "efficiently, flexibly, and accurately" highlights the thinking and decision-making students do through SMPs. As students solve problems in this standard they are

deciding how to make use of structure (SMP 7) and repeated reasoning (SMP 8) while they make sense of the problems they are solving (SMP 1) and reason about the quantities involved and their relationship (SMP 2), and especially using appropriate tools, in this context strategies, strategically (SMP 5). The change in the standards toward efficient, flexible, and accurate strategies, paired with student discourse and collaborative learning, amplifies the revised standards to center the SMPs in daily teaching and learning.

The new domains in WA Math 2026 facilitate teaching and learning through related concepts and multiple on ramps to core big ideas of a grade, actualizing the opportunity to elevate the SMPs through interconnected math standards and data science inquiries.

Opportunities that connect mathematics to relevant and meaningful situations enable students to connect learning to their daily lives, fuel curiosity, and in so doing, deeply entrench the SMPs in the process of math learning.

Standards for Mathematical Practice in Action

The Standards for Mathematical Practice (SMPs) work best when everyone is sharing ideas, whether students are collaborating with each other or with the teacher. Explaining thinking out loud is powerful for learning. Mathematically Productive Instructional Routines (MPIRs) are strategies that get students talking about math and sharing how they think. These routines help teachers understand what students know and give students a chance to learn from one another. They also build a classroom culture focused on collaboration and deeper understanding.

Because MPIRs rely on students explaining and discussing their ideas, they support both math practice standards (SMPs) and language development (WIDA). Students explain their thinking, share solutions, justify their reasoning, and evaluate others' ideas.

As students share their thinking, teachers can check for understanding by comparing student responses to the learning goals and decide what to teach next.

When instructional practices create space for students to demonstrate or explain their thinking to others, then educators can formatively assess student understanding to support Universal Design for Learning (UDL). UDL is a process of lesson design that anticipates and removes barriers for all students to access content and/or demonstrate their understanding of content. The interconnected standards and focus on the SMPs create on ramps for students to use connected standards to access a big idea.

Example

Students in an Integrated Math 1 class are learning about exponential functions. In this example, the students live in a community that was sprayed for spongy moths in the previous year. Students are working with the county to collect data that shows the population and areas of density for the invasive species. As students model their data (through mathematical modeling and the data science inquiry process) students are interacting with a number of standards like, functions, algebra, geometry, and statistics standards (through domain and range, interpreting functions, understanding key features of the function on a graph, distinguishing between models and fitting

functions to data sets, assessing geographic spread and area, and interpreting data).

The assessment of all this learning does not rely exclusively on procedural understanding of algorithms or formulas. Rather, assessment in this context relies on how students are communicating their problem solving and iterative thinking toward a conclusion or justification. In this context, UDL considers some students may have better access to content if the data set is loaded into an interactive online data interface, where they can manipulate and adjust variables to find structure and patterns in the data, where some students may have better access to spreadsheets where they can create formulas to determine spread rates of the spongy moth in different areas. Additionally, UDL considers students may need manipulatives and tangible models to access and demonstrate their understanding of spread of the spongy moth to come to a conclusion or justification. This instructional activity is rich in content, rigorous in learning, connected to the students' lives and community, and takes the learning of the grade and connects it with civic engagement in partnership with the county. In this way, the SMPs, UDL, and WIDA standards amplify math as a tangible, meaningful, and joyful part of their K–12 experience

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