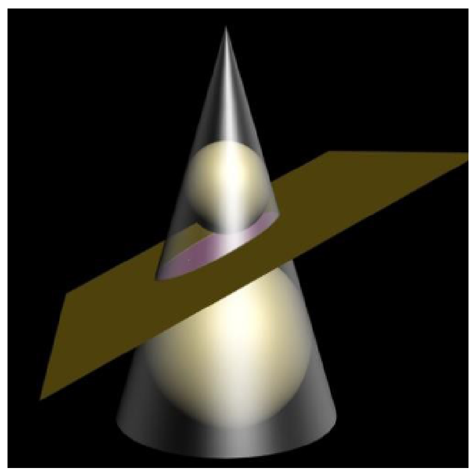
Washington State Bridge to College Mathematics Course



Adapted from Math Ready
A Southern Regional Education Board Transition Course

BRIDGE to COLLEGE

This course is dedicated to Katy Absten, former Math Specialist at OSPI in Washington State. Katy was one of the first people to work on this project and was a true visionary in its creation and implementation. Her work was instrumental in shaping all components of the course – the curriculum, the professional learning structures, and the partnerships. She believed strongly in empowering math students across the state of Washington to learn mathematics and transform their lives in the process.

This book is adapted primarily from the excellent Math Ready materials created by the Southern Regional Education Board (SREB) which are licensed under the Creative Commons Attribution- NonCommercial License. We express our gratitude to SREB for their work on the transitioning to college and career readiness courses, without which the current book would not be possible.

We also thank the wonderful contributions from the following sources:

Dan Meyer Three Acts of a Mathematical Story
varied Creative Commons licenses
Illustrative Mathematics
Creative Commons Attribution-NonCommercial-ShareAlike license
Mathematics Assessment Project
Creative Commons Attribution-NonCommercial-NoDerivs license

Cover images are licensed under the Creative Commons Attribution: NonCommercial License

Pi: The Transcendental Number by Tom Blackwell https://www.flickr.com/photos/tjblackwell/
Vegetable meets Mathematics by anroir https://www.flickr.com/photos/anroir/
Conic section ellipse by Electric Eye https://www.flickr.com/photos/sightrays/
Fractal Spiral 1 by Rosemarie s.w. https://www.flickr.com/photos/rosemariesw/

The initial course design work was supported by a grant from Rockefeller Philanthropy Advisors (https://www.rockpa.org).

Course development and implementation has been supported by a grant from College Spark Washington (http://www.collegespark.org).

July 2017 (updated June 2020; July 2021, July 2022)

Table of Contents

Course Introduction

With the advent of the Smarter Balanced assessment system in 2014-15, Washington began providing assessment results that can inform students, families, and teachers of students' strengths and weaknesses in English and math relative to state learning standards.. With the high school assessment now being administered to sophomores rather than juniors, most students should enter their senior year with the awareness of their capacities in mathematics. Many, if not most, of these students are interested in post-high school opportunities that don't involve a calculus pathway to a STEM-related career. What is the best senior year math option for these students; what are the crucial math skills and abilities they need to develop in their senior year to be prepared for non-calculus pathway college level mathematics courses and the challenges they will face in their first year of higher education?

In Fall 2013, high school and higher education faculty from Washington state began meeting to answer these questions. They started by identifying what it means to be college ready in mathematics for non-calculus pathway courses. Using previously developed college readiness outcomes, the Common Core State Standards (CCSS), and newly generated student profiles of college readiness, faculty from across the K-16 system developed the Bridge to College Mathematics Transition Course Standards which include both content standards and the Standards for Mathematical Practices from the CCSS. Through multiple conversations with a wide range of participants, these course standards were finalized by Spring 2014.

Having settled on course standards, faculty and curriculum designers from K-12 and higher education started the process of developing the Bridge to College Mathematics curriculum. Participants considered the possibility of building an all new curriculum, but since other states and regions had already developed a great deal of successful curriculum, they decided instead to examine the available models. Numerous open resource courses from the Southern Regional Education Board, University of Texas Dana Center, Kentucky Department of Education, Tennessee Board of Regents, Virginia Department of Education and West Virginia Department of Education and the Higher Education Policy Commission were examined, evaluated, and reviewed. The SREB Math Ready curriculum was selected by K-16 educators through a rigorous review process involving a rubric based on the course standards, the Common Core State Standards and the NCTM Mathematics Teaching Practices. (NCTM, 2014).

In the summer of 2014, interested high school teachers and college faculty met to develop a deep understanding of the course and its alignment to the BTCM course standards and NCTM Mathematics Teaching Practices. Project leadership also participated in five days of SREB Math Ready teacher training at the High Schools that Work conference to gain insight into the course as well as the challenges of implementing a college readiness course statewide.

During the 2014-2015 school year, sixteen high school teachers across the state piloted the course in their high school classrooms. In addition, two community college faculty used units from the curriculum in their pre-college mathematics courses. Pilot teachers and students provided extensive feedback on the modules via online forums, face to face regional meetings, and telephone interviews with researchers. In the spring of 2015, a team of pilot teachers, community college faculty and instructional experts reviewed all feedback and revised lessons and units to improve alignment to the BTCM course standards and the NCTM Mathematics Teaching Practices.

Additional resources such as curriculum guides, practice sets, and assessment banks were also developed to support teachers in implementing the course.

After being piloted and refined, both Bridge to College courses (Mathematics and English) were finalized and offered in full during the 2015-16 school year. In that first full year of implementation for the courses there were 74 districts, 114 high schools, and 210 teachers total (106 math) offering the courses, serving almost 4000 students (based on fall enrollment numbers from OSPI). Despite the challenges dealing with the COVID-19 pandemic, the Bridge to College project completed its sixth full year in the 2020-21 school year, with almost 300 teachers serving in 120 districts and 197 high schools. More than 15,000 students statewide have enrolled in Bridge to College Mathematics since the course was developed in 2014.

Educators from across the Washington state K-16 system have worked hard to create the opportunity for students to see themselves as college students and, more importantly, develop the skills, abilities, and mindset necessary to succeed when they get to college. To date students and teachers have consistently reported that the course successfully meets these demands, and the Bridge to College Mathematics course continues to be refined through the ongoing work of the teachers and teacher- leaders involved in the course.

While a great deal of work has been done to develop this course and its supporting documents, there is no question that the most important work of this project begins and ends with teachers and students in the classroom. Students may use their grade of "B" or better in Bridge to College for placement into non-calculus pathway college level math courses at all 34 of Washington state's community and technical colleges as well as at Eastern Washington University, without the need to take a placement test or provide other test scores or documentation. In doing so, the Washington state K-16 system is placing great trust in Bridge to College Mathematics teachers and students.

Teachers will not be alone in their work with this course, however. The Bridge to College project contains the structure for a powerful learning community support system. The BTCM Communities of Practice connect high school teachers, college faculty, and instructional experts in an ongoing, regional partnership to foster authentic learning for all participants. In these regional teams, Bridge Course Leaders and local course teachers meet throughout the year to share their experiences with the course and facilitate collaborative learning. These communities provide ongoing support for teachers to improve their craft and increase student achievement of the course outcomes. While the outcomes, principles, and curriculum are powerful, there is no question that the Communities of Practice are the key element in the evolution of the course and the students it serves. The materials contained in the course provide the foundation for teachers in these Communities of Practice, and their students, to engage in complex, meaningful learning that will prepare all students for the college and career challenges they face immediately after high school.

National Council of Teachers of Mathematics. (2014a). *Principles to Actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics.

Course Name, Code and Description

Bridge to College Mathematics is a year-long course focusing on the key mathematics readiness standards from Washington State's K-12 Learning Standards for Mathematics (the Common Core State Standards, CCSS-M) including intentional development of the eight Standards for Mathematical Practices. The course is designed to prepare students for entrance into non-calculus pathway introductory college level mathematics courses. The course addresses key learning standards for high school including Algebra I, statistics, geometry, and Algebra II standards essential for college- and career-readiness.

Course Name and Code

Bridge to College Mathematics - #WA0003

Course Description

The course curriculum emphasizes modeling with mathematics and the Standards for Mathematical Practice found within Washington K-12 Mathematics Learning Standards (the Common Core State Standards, CCSS-M). Topics include building and interpreting functions (linear and exponential), writing, solving and reasoning with equations and inequalities, and summarizing, representing, and interpreting data. The course is designed to focus on building conceptual understanding, reasoning and mathematical skills and provides students engaging mathematics that builds flexible thinking and a growth mindset. For seniors who are successful in this course (B or better), the *Bridge to College Mathematics* course offers guaranteed[1] placement into a college-level course when entering college directly after high school.

This course must be taught using the ***Bridge to College Mathematics curricular materials*** *and the appropriate course name, and course code. It is required that Units A-G are taught during the school year. Unit H is optional.*

All teachers who are teaching the course for the first time must participate in the year-long professional learning program described later in this document. Teachers returning to the course have ongoing professional learning requirements, also described later in this document.

Transition Course Standards

This 12th grade math college readiness/transition course is designed primarily for students preparing to enroll in college math pathways that don't include calculus. The course content of this course includes the following content standards of the CCSS. There is also an expectation that the Standards for Mathematical Practices will be embedded throughout the course.

Geometry (7-G)

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing

Ratios & Proportional Relationships (7-RP)

Analyze proportional relationships and use them to solve real-world and mathematical problems.

3. Use proportional relationships to solve multistep ratio and percent problems.

The Number System (7-NS)

Apply and extend previous understandings of operations with fractions.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
3. Solve real-world and mathematical problems involving the four operations with rational numbers.

The Real Number System (N-RN)

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $\left(5^{1/3}\right)^3 = 5^{(1/3)\cdot 3}$ to hold, so $\left(5^{1/3}\right)^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Quantities (N-Q)

Reason quantitatively and use units to solve problems

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.

Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context. ★
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.
2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
    a. Use the properties of exponents to transform expressions for exponential functions.

       For example the expression 1.15^t can be rewritten as $\left(1.15^{1/12}\right)^{12t} \approx 1.01212^t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Expressions & Equations (7-EE)

Use properties of operations to generate equivalent expressions.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Expressions & Equations (8-EE)

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

Creating Equations (A-CED)

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions.
12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-plane.

Functions (8-F)

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
3. Interpret the equation y=mx+b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Interpreting Functions (F-IF)

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its

domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
    a. Graph linear functions
    b. Graph exponential and logarithmic functions, showing intercepts and end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the properties of exponents to interpret expressions for exponential functions.
    For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions (F-BF)

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.★
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear and Exponential Models (F-LE)

Construct and compare linear and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Statistics & Probability (7-SP)

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
    b. Informally assess the fit of a function by plotting and analyzing residuals.
    c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Conditional Probability & the Rules of Probability (S-CP)

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

Standards for Mathematical Practice

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.

Washington's K–12 Social Emotional Learning Standards and Benchmarks

Self	Social
Standard 1: *Self-Awareness* – Individual has the ability to identify and name one's emotions and their influence on behavior.	**Standard 4**: *Social Awareness* – Individual has the ability to take the perspective of and empathize with others from diverse backgrounds and cultures.
Benchmark	Benchmark
1A – Demonstrates awareness and understanding of one's emotions. 1B – Demonstrates knowledge of personal strengths, areas for growth, culture, linguistics assets, and aspirations. 1C – Demonstrates self-awareness and understanding of external influences, e.g., culture, family, school, and community resources and supports.	4A –Demonstrates awareness of other people's emotions, perspectives, cultures, languages, histories, identities, and abilities. 4B – Demonstrates an awareness and respect for similarities and differences among community, cultural and social groups. 4C – Demonstrates an understanding of the variation within and across cultures.
Standard 2: *Self-Management* – Individual has the ability to regulate emotions, thoughts, and behaviors	**Standard 5**: *Social Management* – Individual has the ability to make safe and constructive choices about personal behavior and social interactions.
Benchmarks	Benchmark

2A – Demonstrates the skills to manage one's emotions, thoughts, impulses, and stress in constructive ways. 2B – Demonstrates responsible decision-making and problem-solving skills.	5A – Demonstrates a range of communication and social skills to interact effectively with others. 5B – Demonstrates the ability to identify and take steps to resolve interpersonal conflicts in constructive ways. 5C– Demonstrates the ability to engage in respectful and healthy relationships with individuals of diverse perspectives, cultures, language, history, identity, and ability.
Standard 3: *Self-Efficacy* – Individual has the ability to motivate themselves, persevere, and see themselves as capable	**Standard 6**: *Social Engagement* – Individual has the ability to consider others and show a desire to contribute to the well-being of school and community.
Benchmark	Benchmark
3A – Demonstrates the skills to set, monitor, adapt, persevere, achieve, and evaluate goals. 3B – Demonstrates problem-solving skills to engage responsibly in a variety of situations. 3C – Demonstrate awareness and ability to speak on behalf of personal rights and advocacy.	6A – Demonstrates a sense of school and community responsibility. 6B – Demonstrates the ability to work with others to set, monitor, adapt, achieve, and evaluate goals. 6C – Contributes productively to one's school, workplace, and community.

Course Overview

The Bridge to College Mathematics course focuses on the key readiness standards from the Common Core as well as the eight Standards of Mathematical Practices needed for students to be ready to undertake postsecondary academic or career preparation in non-STEM fields or majors. The course addresses standards throughout high school and even earlier, including Algebra I, statistics and geometry, and the Algebra II standards agreed to as essential college- and career-readiness standards for all students, regardless of their intended degree or career path. The full range of content standards found in Algebra II is not addressed because some are not seen as essential college- and career- readiness standards for non-calculus pathway math courses. The course consists of eight units: algebraic expressions, equations, measurement and proportional reasoning, linear functions, linear systems of equations, exponential functions, quadratic functions, and summarizing and interpreting statistical data. While this course covers the basics in math practices and reviews the procedural steps needed to be successful in math, it is designed to be taught in an engaging way based heavily on conceptual teaching and learning. Each unit includes a "hook" at the beginning to engage students and pre-assess prior math experiences and understandings. The hook is followed by several days of tasks that delve deeply into math found in the Standards for Mathematical Practice and the lead headers of the Common Core: focus, coherence and rigor. Each unit also includes at least one formative assessment lesson, allowing the teacher to adapt instruction and learning during the remainder of the unit.

Unit # & Title	Unit Description	Est. # class periods *
Unit A: Building a Culture of Learning Using Data and Statistics	This unit is designed to build an equitable learning environment of curiosity, participation, positive interaction with others, group interdependence and mathematical inspiration. The mathematics contained in the Tasks is intended to be accessible by students at this level and provides multiple points of entry. The outcomes of this unit are intended to be (1) A set of classroom norms based on Standards for Mathematical Practice (MP) and Social and Emotional Competencies (SEL) as well as (2) an increased sense of efficacy and competence for the students because they have had success (3) exploring the mathematics of Statistics and Data. In this unit students will further develop skills to read, analyze, and communicate (using words, tables, and graphs) relationships and patterns found in data sets of one variable. Learning how to choose the appropriate statistical tools and measurements to assist in the analysis, being able to clearly communicate your results either in words, graphs, or tables, and being able to read and interpret graphs, measurements,	24

	and formulas are crucial skills to have in a world overflowing with data. Students explore these concepts while modeling real contexts based on data they collected.	
Unit B: Algebraic Expressions	The algebraic expressions unit is designed to solidify student understanding of expressions while providing the students with an opportunity to have success early in the course. The recurring theme integrated in this unit focuses on engaging students using and expanding the concepts found within purposefully chosen activities. Through guided lessons, students will manipulate, create and analyze algebraic expressions and look at the idea of whether different sets of numbers are closed under certain operations. The writing team selected content familiar to the students in this unit to build student confidence and to acclimate students to the course's intended approach to instruction.	14
Unit C: Equations	The equations unit calls for students to construct and evaluate problems that involve one or two steps while seeking understanding of how and why equations and inequalities are used in their daily lives. Students are also asked to use the structure of word problems and equations to rewrite and solve equations in different forms revealing different relationships.	13
Unit D: Proportional Reasoning	The proportional reasoning unit first revisits multiple methods experienced during middle school (including ratio tables, double number lines, graphs, equations, and unit rates). These methods are then utilized and built upon in addressing concepts that include unit conversions, using proportions for scaling, and calculating probabilities. The unit requires higher- order thinking and number sense in order to get to the true intent of the standards covered.	10
Unit E: Linear Functions	The linear functions unit takes students back to the foundation of all high school mathematics; an in-depth study of linear functions. Along with allowing students to differentiate between relations that are functions and those that are not, the unit helps students specifically examine characteristics of linear functions. By looking closely at linear functions in multiple forms, students are expected to graph and write equations, as well as interpret their meaning in context of the slope and y-intercept. Additionally, students analyze linear patterns and	19

	correlation in bivariate scatterplots. Students conclude with a project allowing them to collect their own data and write a line of best fit from that data.	
Unit F: Linear Systems of Equations	The systems unit deals with solving systems of linear equations. This involves helping students classify solutions (one, none or infinitely many), as well as set up and solve problems using systems of equations. This unit also asks students to choose the best way to solve a system of equations and be able to explain their solutions.	11
Unit G: Exponential Functions and Introduction to Logarithms	The exponential unit develops fluency in exponential functions through varying real-life financial applications/inquiries. The unit builds student understanding of these higher-level functions and gives them the opportunity to reflect upon the ramifications of their future financial choices. Basic logarithmic operations are included as a means to solving exponential equations.	18
Unit H: Quadratic Functions **OPTIONAL**	This unit is an expansive look at quadratic functions: their graphs, tables and algebraic functions. It stresses multiple approaches to graphing, solving and understanding quadratics, as students explore, make conjectures and draw conclusions in group-work settings. The unit assumes students have seen quadratics before but may not have a concrete, transferable understanding of quadratic functions. The unit does not cover algebraic manipulations (multiplying and factoring), as these are in earlier units.	27

* Estimated number of class periods is based on a 50 minute class period, and may need to be adjusted to fit time available per day. Teachers should use ongoing formative assessment to decide how much time should be spent on each lesson in a unit and where, in each unit, additional practice time may be necessary.

Recommended Priority for Student Enrollment

The *Bridge to College Mathematics (BtCM)* Course is a math course designed primarily for students interested in college programs/areas not requiring calculus and who may need some additional work to be prepared for college-level coursework in math. Enrollment priority should be given to:

- Seniors who have taken a 3rd-year math course (Algebra 2, Integrated III, or other algebra-based 3rd credit alternative) and would benefit from additional math intervention to be prepared for college math.
- Seniors who are recommended by high school instructors based on other factors such as readiness and their high school and beyond plans.

Important Notes:

1. BTCM qualifies as one of the designated options for the “multiple pathways to graduation” component of the high school graduation requirements as defined by the 2019 Legislature (https://www.k12.wa.us/student-success/graduation/graduation-requirements/hb-1599-multiple-pathways-graduation).
2. BCTM ***can qualify*** as a ***3rd credit of math*** if the student has already attempted Algebra 2 or is credit-deficient.
3. Currently, the Bridge to College Mathematics Course does not qualify for NCAA. It should be noted that the course is intended as a senior-level, 4th-year course in mathematics but the NCAA only requires 3 years of math, so not qualifying as a 'core course' for the NCAA should not be a problem for most students.
4. **Baccalaureate Requirements:** To meet the minimum admissions requirements for state baccalaureate institutions, students need to pass Algebra 2 for their 3rd credit of math. The Bridge to College Mathematics Course does meet the baccalaureate senior year requirement for a math or quantitative reasoning course as determined by the Washington Student Achievement Council (College Academic Distribution Requirements (CADR), 2014).

Washington State Automatic Placement Agreement

As part of the implementation of the Washington State Learning Standards and Smarter Balanced Assessment, the Washington community and technical college system has approved an automatic placement agreement for high school students. All 34 colleges in the system offer high school students the opportunity to use their scores on the high school Smarter Balanced assessment to establish their readiness for college-level coursework.

The goal of the agreement is to increase the number of students enrolling directly into college courses without remediation by

A. offering students an early opportunity to know whether they are ready for college-level academic work;
B. providing an incentive for achieving the Common Core standards as reflected in the Smarter Balanced assessment; and
C. creating alternatives for students, if necessary, to use their last two years of high school more effectively in getting ready for college-level work.

Based on changes in the high school assessment system, the original agreement was updated beginning with students who took the high school assessment in spring 2018 and is in effect for the high school graduating classes through the Class of 2022. During the Fall 2022 quarter we'll be asking our key community and technical college system groups (placement staff, counselors/advisors, deans, academic vice-presidents) to review and make decisions about the existing agreement. We believe that the courses are now so well-established that the agreement should be extended indefinitely; given the challenges of the past several pandemic years, I'm very optimistic that at a minimum they will agree to extend the agreement for a few years to allow us to collect additional data on how well the courses are serving students. For now, you can tell students in this year's Bridge to College classes (Class of 2023) that we expect the agreement to be in place for Fall 2023; we 'll have a formal update for you--and news about the status of the agreement beyond Fall 2023--by January 2023.

BRIDGE TO COLLEGE MATHEMATICS

Automatic Math Placement Options Available to Students Entering Directly from High School:

MATH

For placement into ***Math& 107*** *(Math in Society),* ***Math& 146*** *(Statistics), or their equivalents:*
 	
1. Level 3 or 4 score on high school Smarter Balanced assessment **plus**
    a. B or better in Algebra 2, and
    b. successful completion (passing grade) of one math course in the junior or senior year
    OR
2. B or better grade in designated *Bridge to College Math* class as a senior
 	
For placement into ***other entry-level math courses (including pre-calculus):***
 	
1. Requires Smarter Balanced Level 3 or 4 **plus**
2. B or better in a high school pre-calculus or higher course

NOTES:

1. Students interested in enrolling in **dual-credit courses** (Running Start, College in the High School) as juniors can use their discipline-appropriate Smarter Balance scores to enroll in ***Math& 107*** *(Math in Society),* ***Math& 146*** *(Statistics), or their equivalents* (requires score **plus** B or better in Algebra 2).
2. For all score levels in math, placement into more advanced courses than designated in the agreement will depend on additional local institutional placement processes (transcript, high school GPA, additional testing, etc.).
3. The Bridge to College courses are not currently available statewide; for a list of schools/districts offering the courses see the Bridge to College page on the State Board for Community and Technical College web site. On student transcripts the courses can be identified by their common course codes (WA0001 for English, WA0003 for math).

Please contact Bill Moore (360-704-4346, bmoore@sbctc.edu) if you have any specific questions.

Assessment Practices and Resources

As part of the placement agreement with the Washington community and technical college system (and Eastern Washington University), a student who receives a B or higher in the Bridge to College Math transition course in their senior year of high school will be entitled to placement into entry level college math courses that are not on the calculus pathway (primarily MATH& 107--Math in Society--and MATH& 146--Statistics). The automatic agreement only applies if they enroll at a participating institution in the fall quarter immediately following graduation from high school, although some colleges may choose to extend that timeframe to a full year. To qualify for the placement agreement the student must complete the full year-long course and must earn a B or higher in the second semester of Bridge to College Mathematics.

Grading (assessment) is a complex and difficult endeavor, and practices vary dramatically between classrooms, schools, and districts. There is no required specific assessment for all students in the course across the state; teachers are expected to assess their students appropriately throughout the year on the course standards, using high-quality classroom-based assessments that offer students a range of opportunities to demonstrate mathematical knowledge and skills. Overall grades in the course can be determined by many components (homework, participation, group exercises, writing, online practice, etc.), but a significant portion of the overall grade should come from assessments aligned to the course standards. Assessment resources for each unit are provided. Instructors should use these resources to create various types of end of unit assessments (tests, application tasks, projects, portfolios, reflections). Proctored exams should be included in the assessment process in order to prepare students for college math courses.

A “B” in this course should represent a student’s understanding of the mathematics of the course as identified by the course standards, including the Standards for Mathematical Practices. The overall expectation is that students who earn a grade of B or higher in the course should be prepared for the previously mentioned non-calculus pathway college level math courses.

As part of the required professional learning in Bridge to College Math, new teachers participate in a series of Community of Practice (CoP) meetings during the school year. To help create a shared understanding of the “B or better” performance expectations, teachers will collaboratively choose at least one common assessment to give their students prior to each of these meetings. Every CoP meeting will include a time where teachers work through a protocol of looking at student responses to this assessment, discussing which examples represent B level students and which do not. Past results of these discussions have been analyzed and used to create assessment rubrics and anchor tasks that are included in the teacher resources. This collection of student work will be revised as CoPs continue to have these conversations about college readiness. The outcomes of the CoPs will continue to be shared with the Bridge to College Mathematics community, which includes math faculty from Washington community and technical colleges as well as high school math teachers.

Course Supplies

The Bridge to College Mathematics course is designed to be taught in an engaging way based heavily on conceptual teaching and learning. This type of course requires that classrooms be equipped with supplies and materials that may not be normally purchased for a high school mathematics classroom.

Classroom Supplies List

This list includes special materials that might not normally be found in a high school mathematics classroom. It does not list materials such as graph paper, calculators, rulers, etc., as it is assumed these are part of a normal high school mathematics class. It also does not include any student handouts from the student or teacher manual that may need to be printed, according to individual teacher preference.

Unit A - Building a Culture of Learning Using Data and Statistics

- Roll of brightly colored twine, string, or yarn
- Marshmallow Challenge kits: 20 sticks of spaghetti, 1 yd masking tape, 1 yd string, 1 marshmallow.
- Measuring tape
- Timing device
- Chart paper & Markers
- Internet to access videos and website
- Tools for making statistical graphs (protractors, rules, graph paper, etc.)
- Spinners, coins or marked chips, dice
- 1 copy of Card Sets: *Frequency Graphs* and *Interpretation*s per group

Unit B - Algebraic Expressions

- Chart paper & markers
- Square color tiles (optional)
- 6-sided die
- Mini whiteboards (optional, but recommended for the entire course)
- 2 sets “I have/Who has” cards

Unit C - Equations

- 1 set of 6 Equation Cards & 12 index cards per group
- 1 *Card Set*: *Always, Sometimes, or Never True?* per group
- Chart paper & markers
- Mini-whiteboards
- Colored Pencils

Unit D - Proportional Reasoning

- Chart Paper & markers
- Book: If You Hopped Like a Frog (ISBN-13: 978-0590098571)
- Screen and projector to show video
- Set of “If You Hopped Like a Frog” statement cards, cut out
- Sticky Notes
- Rulers
- Random items to use as a non-standard measurement unit
- Heart Rate Problem Cards
- Clock or stop watch to measure pulse rate
- Data from student survey in Unit A

Unit E - Linear Functions

- Function/Not Function cards (1 set per pair of students)
- Matching Equations cards (1 set per pair of students)
- Graphing Linear Equation in Context cards (2 sets per class)
- Activity Cards (1 set per four students)
- Materials for activity: Water balloons, rubber bands (same size), measuring device (tape measure), masking tape
- Access to internet for videos

Unit F - Linear Systems of Equations

- Materials for lesson 1 depend on implementation style but may include chart paper, markers, yard sticks, different colors of yarn, and/or masking tape
- 1 copy of *Card Set A*: *Equations, Tables & Graphs* and two cut up copies of *Card Set B*: *Arrows* per group
- Poster board

Unit G - Exponential Functions and an Introduction to Logarithms

- Access to internet for student research
- 1 copy of *Card Set: Investment Plans* per group
- 1 copy of *Card Set: Formulas* per group
- 1 copy of *Card Set: Graphs* per group
- 1 copy of *Card Set: Tables* per group
- 1 copy of *Card Set: Statements* per group
- 1 copy of *Card Set: WAR* per pair

- Poster paper & markers

Unit H - Quadratic Functions (Optional)

- Lesson 1: Each group should have access to the following supplies: tongue depressors, gummy bears, rubber bands (same size), index cards, chart paper and markers
- Mini-whiteboards and markers
- Domino Cards
- Algebra tiles
- Marshmallows
- Timing device
- Internet to access videos and website

Online Course Resources and Support

The Bridge to College Mathematics course uses CANVAS as a platform for distributing course materials and building a network of virtual support for teachers, team leaders, and course trainers. All Bridge to College Mathematics teachers should be automatically enrolled in the updated CANVAS course in August of the upcoming school year..

Course Materials

Printable copies of all course materials are posted on CANVAS for teachers to download and use to create student workbooks, handouts, Power Points, etc. Unit level instructional materials include:

- Curriculum Guide
- Teacher Manual
- Student Manual
- Assessment Bank
- Assessment Bank Answer Key
- Rubrics and Anchor Tasks for selected tasks

Curriculum Updates as of August 2022

Rather than being a standalone unit(s), statistics content has been embedded into existing units. In addition, unit labels have been changed to letters (A - H) to minimize confusion from previous years' versions of the course material. Additionally, most instructional activities are now labeled as numbered tasks, which means many task numbers have changed.

Virtual Network

To facilitate the development of a network of virtual support for course teachers, team leaders, and course trainers, you will find discussion forums on CANVAS. In addition, the Bridge Course Leadership Team has developed various methods of virtual collaboration for BTC teachers, including summer and school year workshops and regional communities of practice.

Other Resources

Each Bridge Course teacher is entitled to download a copy of the pdf of NCTM's Principles to Actions: Ensuring Mathematical Success for All which is posted in CANVAS. Please note that while the e-books remain the property of the participants to whom they are distributed, they are not to be shared further as a matter of both copyright and common courtesy.

Professional Learning and Support System: New Teachers

It is critical that all teachers and leaders involved in providing the Bridge to College Math course to students are well prepared in the course content, delivery expectations, and regional support network available. To support a strong statewide implementation, each new teacher is required to participate in the equivalent of 33 hours of professional learning and networking over the course of the year (13 in the summer, 20 during the school year).

All new teachers will attend an initial 2-day summer institute (13 hours). The goals of the summer institute:

- Understand the goals of the course and the expectations of teaching the course
- Understand the depth of the mathematics in the curriculum and the three aspects of each unit: hook lessons, tasks, and formative assessment lessons
- Gain a deeper understanding of Units A and B in preparation for implementation in the fall.
- Make connections to the CCSS-M SMPs, where it makes sense (teaching practices, college readiness, etc.)
- Understand the 8 Mathematics Teaching Practices; focus and reflect on school & individual practices related to facilitating meaningful mathematical discourse and supporting productive struggle.
- Access the Bridge to College Math Canvas site & become a member of the statewide network.

New teachers are then organized into ***Communities of Practice (CoPs)*** to engage in 20 additional hours of professional learning:

- The CoPs will meet 7 times (approximately once per month) during the school year to support each other and focus on a strong implementation of the course statewide.
- Meetings will be a combination of in-person (two days per year), synchronous virtual, and asynchronous virtual.
- The focus of the meetings will be on sharing successes and challenges with others, learning more about upcoming units, and examining student work.

Month	Hours	Modality	Focus
Sept	2	Virtual synchronous	Connecting as a Community Sharing Successes and Challenges Preparing to Teach Unit C
Oct/Nov	5	In-person	Preparing to Teach Units D and E What is a B? Sharing Student Work
Dec	2	Virtual asynchronous	Equity Article Discussion Survey and Reflections on Units

Jan	2	Virtual synchronous	Reconnecting as a Community Sharing Successes and Challenges Five Practices
Feb/Mar	5	In-person	Preparing to Teach Units F and G What is a B? Sharing Student Work
April	2	Virtual asynchronous	Disrupting Unproductive Mindsets Discussion Survey and Reflections on Units
May	2	Virtual synchronous	Reconnecting as a Community Sharing Successes and Challenges Next Steps for Students/Teachers/Districts

Total Hours = 20

New teachers are required to participate in both the New Teacher Institute and at least 15 hours of the school year professional learning meetings. Once they have completed all these requirements and taught the course for a school year then they will be considered a Returning Bridge to College Math Teacher.

Professional Learning and Support System: Returning Teachers

Each returning teacher is required to choose one of several provided options of professional learning that is equivalent to five hours between September 1 and August 31 in order to be eligible to teach in the following academic year. Options will be communicated as they are scheduled and made available. Some options are in-person and some are virtual.

Philosophy of Mathematics Learning and Teaching

Vision of Mathematics Education

In July 2011, Washington adopted the Common Core State Standards for Mathematics (CCSS-M) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) as the new Washington State K–12 Learning Standards for Mathematics. These standards replaced the state's 2008 Mathematics Learning Standards. The Washington State K–12 Learning Standards for Mathematics are built on an intentional progression of the skills and knowledge necessary for all students to be ready for career, college, and life when they exit high school. The progressions of learning provide specific focus for each grade level. The standards lay the groundwork for this vision of mathematics that better fits the skills students need to be productive members of society.

Building on the work of the National Council of Teachers of Mathematics (NCTM), the vision of mathematics education requires students to be problem solvers, to reason quantitatively and to understand and analyze data. Previously, mathematics programs emphasized computation and memorization. Today, students not only need to be fluent and flexible with numbers and operations, students need the capacity to apply concepts and skills to novel situations, to approach real-world problems with stamina, and to understand that there may be multiple viable solution paths and solutions, depending on the context of the problem and the assumptions of the problem-solver.

Success in mathematics is not reserved for an elite few

A key component of the Washington State K–12 Learning Standards for Mathematics (WA State Standards) are the Standards for Mathematical Practice. These standards reflect a key shift in mathematics education and describe the expertise that mathematics educators at all levels should seek to develop in their students. The Standards for Mathematical Practice are:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Mathematics instruction, then, should use the mathematical practices to engage students in the mathematics content and develop students as "practitioners of the discipline of mathematics." For more information on the Standards for Mathematical Practice, see http://www.corestandards.org/Math/Practice/

Additionally, the mindset that success in mathematics is reserved for an elite few contradicts mathematics educational research. Many adults assume that differences in mathematics performance reflect differences in innate ability, rather than differences in individual effort or opportunities to learn. These expectations profoundly underestimate what students can do. The basic principles, concepts, and skills of mathematics are within reach of all students. When parents and teachers alike believe that hard work pays off, and when mathematics is taught and learned by using the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency, mathematics performance improves for all students.

Research has demonstrated that mathematical proficiency is an obtainable goal. (The National Academies, p. 30) It is our duty, therefore, to authentically engage all students in the discipline of mathematics as a foundation for reasoning quantitatively, solving rich problems, and analyzing data to make meaning of information and gain proficiency in analyzing and solving problems.

Focus, Coherence, and Rigor

The Washington State K–12 Learning Standards call for shifts in the way we approach mathematics education. The shifts are:

- Greater focus on fewer topics
- Coherence: Linking content and thinking across grades
- Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity

"Focus" means deep engagement with the major work within each high school course. Rather than racing to cover many topics superficially, the standards ask mathematics teachers to deepen the way time and energy are spent on fewer, key math concepts. "Coherence" requires that content be carefully connected across high school courses, intentionally building on prior knowledge. "Rigor" refers to deep understanding of mathematics concepts. Students must have the opportunity to access concepts from multiple entry points and perspectives. Students must also be fluent with calculations and procedures so they can access more complex concepts and procedures. Finally, students must have the opportunity to apply concepts and procedures to novel situations (Common Core State Standards Initiative, 2015).

Mathematical Representations and Manipulatives

Instruction at all grade levels should incorporate the progressive use of concrete manipulatives, representational models, and abstract symbols (Forbinger & Fuchs, 2014). Much of traditional mathematics instruction focuses on computation and students' ability to apply procedures quickly and accurately. According to the National Council of Teachers of Mathematics (NCTM), procedural fluency, however, includes "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another" (NCTM, 2014b, p.1). This definition of procedural fluency pushes the bounds of traditional mathematics instruction, as it requires foundational knowledge of concepts, reasoning strategies, properties of numbers and operations, and problem-solving methods (NCTM, 2014b). The rigor of the state standards includes balancing conceptual understanding, procedural fluency, and problem solving. Instruction, then, must be balanced to address the mathematics content and practice standards through a variety of approaches.

The use of models or representations to manipulate and communicate about mathematical ideas supports students in making connections among mathematical ideas, understanding computations and procedures, and solving problems. The more ways that students have to think about and test ideas, the better their ability to integrate them into their current conceptual understanding to develop a deep relational understanding. "Strengthening the ability to move between and among representations improves students' understanding and retention of ideas" (Van de Walle, 2013, p. 22).

Mathematical representations can include words, manipulatives, pictures, models, diagrams, equations, and tables and graphs of functions and relationships.

Mathematics Teaching Practices

In 2014, NCTM published a book, *Principles to Actions: Ensuring Mathematical Success for All*. The principles in this text represent "strongly recommended, research-informed actions for all teachers, coaches, and specialists in mathematics" (NCTM, 2014a, p. 4) including any interventionists who will be working to assist children in their mathematics study. These eight mathematics teaching practices reflect the range of instructional strategies and approaches necessary to promote deep learning of mathematics.

1. **Establish mathematics goals to focus learning.**
   "Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions" (NCTM, 2014a, p. 12).
2. **Implement tasks that promote reasoning and problem solving.**
   "Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014a, p. 17).
3. **Use and connect mathematical representations.**
   "Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving" (NCTM, 2014a, p. 24).
4. **Facilitate meaningful mathematical discourse.**
   "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014a, p. 29).
5. **Pose purposeful questions.**
   "Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014a, p. 35).
6. **Build procedural fluency from conceptual understanding.**
   "Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems" (NCTM,2014a, p. 42).
7. **Support productive struggle in learning mathematics.**
   "Effective teaching of mathematics consistently provides students, individually and collectively, with

opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships” (NCTM, 2014a, p. 48).

8. **Elicit and use evidence of student thinking.**
   “Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning” (NCTM, 201a4, p. 53).

References

Common Core State Standards Initiative. (2015). Key shifts in mathematics. Retrieved from http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

Forbinger, L. L., & Fuchs, W. W. (2014). *RtI in Math: Evidence-based interventions for struggling students.* New York, NY: Routledge.

National Council of Teachers of Mathematics. (2014a). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014b). Procedural fluency in mathematics: A position of the National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/

The National Academies. (2002). *Helping Children Learn Mathematics.* Washington, D.C.: National Academy Press.

Van de Walle, J. A., Lovin, L. A. H., Karp, K. S., & Bay-Williams, J. M. (2013). *Teaching student- centered mathematics* (Second ed.). Washington, D.C.: Pearson

Boaler, Jo (2016). *Mathematical Mindsets.* San Francisco, CA: Jossey-Bass.

Acknowledgements

This course is the result of an extensive and ongoing collaboration among staff and key personnel from state agencies, school districts and high schools, and colleges and universities. In particular, we deeply appreciate the time investment and substantive contributions from individual high school teachers and higher education faculty in the design, pilot-testing, and refinement of these courses and would like to acknowledge those individuals below. We hope we haven't missed anyone but apologize in advance if we have, or if we somehow managed to get a name wrong; we also recognize individuals may have moved or changed jobs, so what's listed shows their affiliation when they were connected to this work. If you see corrections or additions let us know and we'll update the list as soon as we can!

Bill Moore, State Board for Community and Technical Colleges (SBCTC)

Rebecca Wallace, Office of Superintendent of Public Instruction (OSPI)

Project Co-Leads

Thanks to former OSPI staff Jessica Vavrus and Amy Vaughn for their work in helping to get the initial course off the ground, and Kathe Taylor for guidance and support in the years since.

Project Leadership Team (Mathematics)

Megan Luce, Math Faculty, Cascadia College (higher education discipline lead)

Arlene Crum, Director of Mathematics (OSPI)

2022-2023 Course Leaders

Casey Bruinsma, Susan Cedar, Meg Chambers, Dawn Draus, Mary Ellen Huggins, Cory Kunz, Kristianna Johnsen, Leslie Nielsen, Debbie Olson, Linda Richard, Cate Roscoe, Taya Schweizer, Tamara Smith, Becky Spitznas, Grayce Willard

Previous Course Leaders

Greta Bornemann, Jill Brouillard, Autumn Carper, Kim Coatney, Jeff Crawford, Wendy Droke, Anne Gallagher, Deb Gribskov, Stefhany Guadarrama, Shereen Henry, Allie Hernandez Nelson, Coretta Hoffman, Carolyn Johnson, Jamie Ladwig, Micaela Newman, Elizabeth Tubac, Nicole Wethall

BRIDGE TO COLLEGE MATHEMATICS

Pilot Teachers

Jennifer Bressert, South Kitsap SD
Amy Hancock, Mead SD
Jeff Cimaomo, Franklin HS
Coretta Hoffman, Mead HS
Michael Coffin, Prairie HS
Kristianna Johnsen, Shorewood HS
Ryan Cornell, Central Valley SD
Carolyn Johnson, Rochester HS
Madelyn Erickson, Olympic HS
Julie O'Brien, Franklin HS
Chris Fugleberg, Kelso HS
Linda Richard, Shoreline SD
Bruce Graham, Grandview SD
Erin Schoenlein, Clark College
David Groat, Battle Ground HS
Mary Wallace, Mead SD

Course Design/Development Participants

Barbara Alvin, EWU
Patrick Kjack, Wenatchee SD
Deann Anguiano, Marysville-Pilchuck HS
Jeremy Koziol, Spokane CC
Emily Asher, Bates TC
Meredith LaFlesh, Tacoma
Pat Averbeck, Edmonds CC
Jennifer Laveglia, Bellevue College
Carol Avery, Tacoma CC
Diana Lee, Highline
Rick Biggerstaff, EWU
Kris Lindeblad, WSU
Sue Bluestein, ESD 112
Michael Lundin, CWU
Greta Bornemann, PSESD
Kelly Lynn, EWU
Helen Burn, Highline CC
Mausumi Maulik, Bellevue College
Chris Carlson, Centralia College
Carlea McAvoy, South Puget Sound CC
Linda Carney, Shadle Park HS
Teresa McComber, North Mason HS
Kelly Casto, Richland SD
Sherry McLean, Lake Washington
Chris Chelin, Kennewick SD
Terry Meerdink, Highline CC
Sandy Christie, Consultant
Laura Moore- Mueller, Green River CC
Sellie Clark, Tacoma
Andrea Motyka, Peninsula CC
Catherine Conway, North Seattle
Lynn Neal, Bates TC
Jackie Coomes, EWU
Michael Nevins, Everett CC
Sandy Cooper, WSU
Ron Noble, WA Applied Math
Curtis Crawford, Columbia Basin College
Troy Oelschlager, Klahowya Secondary
Mike Dodge, Olympic College
De Pham, Franklin HS
Ryan Dorman, South Seattle CC
Emile Pulido, Bates Technical College
Ryan Downing, Grandview SD
Roxane Ronca, WWU
Dawn Draus, Lower Columbia College
Richard Sander, Everett SD
Nicole Duvernay, Spokane CC
Lauren Sandven, Shoreline
Alex Eklund, Seattle PS
Amy Scheller, Kelso HS
Elissa Farmer, Seattle PS
Debra Schneider, Evergreen PS

BRIDGE TO COLLEGE MATHEMATICS

Kendra Feinstein, Tacoma
Laura Schueller, Walla Walla
Frank Fisher, Shoreline SD
Mike Selle, Bridgeport HS
Wes Fitz, Central Kitsap HS
Lee Singleton, Whatcom CC
Vauhn Foster- Grahler, TESC
Terrie Skeie, Lower Columbia College
Jana Freese, Columbia Basin College
Tamara Smith, ESD 114
Nancy Goodisman, Shoreline CC
Darlene Snider, Walla Walla
Melissa Graham, EWU
Rosalie Tepper, Shoreline CC
Deb Gribskov, Kelso HS
Trung Tran, Tacoma CC
Michelle Gruber, Northshore SD
Dana Updegrove, Bellevue College
Carol Hattan, Vancouver SD
Mary Ward, Bates Technical College
MaryKe Haynes, Battle Ground PS
Nicole Wethall, Seattle PS
Shereen Henry, Shoreline SD
Sharon Wiest, Wenatchee Valley CC
Jenny Hughes, Columbia Basin